CHAPTER

4

Fitting a Line to Data

Overview

Chapter 4 emphasizes slope in the context of finding lines of fit. **Lesson 4.1** presents a formula for determining slope using any two points on a line. Students learn about all four slope types: positive, negative, zero (horizontal), and undefined (vertical). In **Lesson 4.2,** students use their understanding of the intercept form to fit lines to data. **Lessons 4.3** and **4.4** introduce students to the point-slope form and its application. In **Lesson 4.5,** students apply their understanding of the point-slope form to fit lines to data. **Lessons 4.6** and **4.7** establish and develop the method for determining lines of fit based on quartiles of the two data sets; this standardized procedure allows everyone to get the same equation to model a given set of data. In **Lesson 4.7,** students compare and evaluate methods of fitting lines to data. **Lesson 4.8** is an activity day for reviewing lines of fit.

The Mathematics

Slope

Previous chapters related the constant rate of change of a data set or equation to the steepness of a line graphing the data or equation. Now we see a definition of the slope of a line through two points, as the ratio of the difference in y-coordinates to the difference in corresponding x-coordinates: $\frac{\textit{change in } y}{\textit{change in } x}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

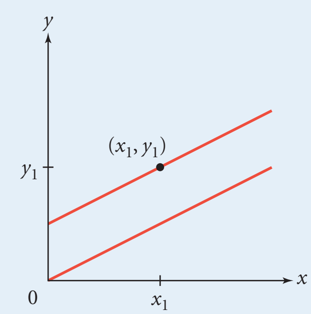
This formula gives a static way to think about slope. A dynamic concept can also be useful. Imagine that you're moving from left to right along the line. Page 219 describes this dynamic concept of slope and gives ideas to help students understand dividing by zero.

Point-Slope Form

The point-slope form of an equation is often given as $\frac{y - y_1}{x - x_1} = m$, where m is the given slope and (x_1, y_1) is a point on the line. This form emphasizes that for "any point on the line" (x, y), the slope of the line segment between that point and the given point is the ratio $\frac{y - y_1}{x - x_1}$.

Using b instead of m to represent the slope, you can write the point-slope form as $y - y_1 = b(x - x_1)$, obtained from the previous form of the equation by multiplying both sides by the denominator. For graphing an equation on a graphing calculator, y must be expressed in terms of x. This motivates the form the student text uses, $y = y_1 + b(x - x_1)$.

You can see the equation $y = y_1 + b(x - x_1)$ as the result of translating the variation $y = bx$ to the right x_1 units and up y_1 units. When you slide up, you add to the y-value. But if you also slide the line to the right, the line now goes through the point (x_1, y_1), and the line will cross the y-axis below y_1.



In practice, the point-slope form (in any version) is more useful than the intercept (or slope-intercept) form. Rarely do you know the y-intercept. If you need to find it, you can substitute 0 for x_1 in the point-slope form, so resist any impulses to change all linear equations into an intercept form.

Lines of Fit

A *line of fit* is intended to represent the points in a data set. You use it when you're trying to make a prediction based on a scatter plot that looks linear. You might use several methods of finding lines of fit. Very common is the *method of least squares,* in which the sum of squares of vertical distances between a line and the data points is minimized. This method is studied in statistics courses and advanced algebra.

In *Discovering Algebra,* students find the slope b of the line between two representative points, graph the variation $y = bx$, and then translate that line to an appropriate y-intercept so that it appears to

represent the data. This gives the intercept form, $y = a + bx$, of a line. Students also get the point-slope form by translating the line $y = bx$ to pass through one of the two points chosen to find the slope.

A third modeling method uses *Q-points*. For each of the four Q-points, the first coordinate is a first or third quartile of the x-coordinates of the data set, and the second coordinate is the first or third quartile of the y-coordinates: (x_{Q1}, y_{Q1}), (x_{Q3}, y_{Q1}), (x_{Q1}, y_{Q3}), and (x_{Q3}, y_{Q3}). These four points form a rectangle. The line of fit lies along one diagonal.

Another method of linear regression—the *median-median method*—is implemented on calculators and might be an appropriate extension for students in this course. In it, you divide each set of coordinates into thirds. You use the medians of the outer thirds to determine a line. Then you shift that line vertically $\frac{1}{3}$ of the way toward the point given by the medians of the middle thirds.

Using This Chapter

Lessons 4.6 and 4.7 provide valuable practice with linear equations and slope. Lesson 4.8 is a very popular activity and can be done even if you have skipped Lessons 4.6 and 4.7; it provides a good summary of Chapter 4. If your state standards require you to cover slopes of parallel and perpendicular lines, you may want to teach Lesson 11.1 directly after Lesson 4.4.

Resources

Discovering Algebra Resources

Teaching and Worksheet Masters
Lessons 4.1, 4.4, 4.8

Calculator Notes 1D, 3D, 4A, 4B

Sketchpad Demonstrations
Lessons 4.1, 4.3, 4.4

Fathom Demonstrations
Lessons 4.2, 4.5, 4.6

CBR Demonstration
Lesson 4.5

Dynamic Algebra Explorations online
Lessons 4.1, 4.6

Assessment Resources
Quiz 1 (Lessons 4.1, 4.2)
Quiz 2 (Lessons 4.3, 4.4)
Quiz 3 (Lessons 4.5–4.7)
Chapter 4 Test
Chapter 4 Constructive Assessment Options

More Practice Your Skills for Chapter 4

Condensed Lessons for Chapter 4

Other Resources

Data in Depth by Tim Erickson.

For complete references to this and other resources, see www.keypress.com/DA.

Materials

- graph paper
- rulers
- uncooked spaghetti
- several books
- 5 oz plastic cups
- string
- pennies (100 per group)
- empty boxes (one per group), *optional*
- sharp scissors or awls
- stopwatch
- bucket or other object to pass
- toy figures
- identical rubber bands (300)
- tape measures, metersticks, or yardsticks
- video camera, *optional*

Pacing Guide

	day 1	day 2	day 3	day 4	day 5	day 6	day 7	day 8	day 9	day 10
standard	4.1	4.2	4.2	4.3	4.4	4.4, quiz	4.5	4.6	4.6	4.7
enriched	4.1	project, 4.2	4.2	4.3	4.4	4.4, quiz	4.5	4.6	4.6, project	4.7
block	4.1, 4.2	4.2, 4.3	4.4	quiz, 4.5	4.6	project, 4.7	quiz, 4.8	review, assessment		

	day 11	day 12	day 13	day 14	day 15	day 16	day 17	day 18	day 19	day 20
standard	4.8, quiz	4.8	review	assessment						
enriched	4.8, quiz	4.8	review, TAL	assessment						

CHAPTER 4 OBJECTIVES

- Learn how to calculate the slope of a line with slope triangles and the slope formula
- Learn about slopes of rising, falling, horizontal, and vertical lines
- Learn the point-slope form of an equation of a line
- Check the equivalence of linear equations by using algebraic properties
- Learn several approaches to finding a line that represents a set of real-world data points (eyeballing, representative points, Q-points)
- Evaluate the results of those approaches

CHAPTER

4

Fitting a Line to Data

Artists, like mathematicians, use lines to summarize their observations. An artist's data include contour, texture, color, shape, motion, and balance. The American artist Romaine Brooks (1874–1970) reduced her entire set of observations into the lines you see in this pencil sketch titled *Departure.*

OBJECTIVES

In this chapter you will

- define and calculate slope
- write an equation that fits a set of real-world data
- review the intercept form of a linear equation
- learn the point-slope form of a linear equation
- recognize equivalent equations written in different forms

You can ask the class to brainstorm other meanings of the word *line*. Some of these are boundary, course, limit, queue, series, and trend. Point out that, in colloquial use, a line is not always straight. In mathematics, *line* usually means a straight line or segment, whereas *curve* refers to a bending of a line.

In visual art, a line might be used in various ways. For example, an *outline* calls attention to the shape; this artist has emphasized the shape the human figure assumes in the position depicted. A *contour* gives the illusion of three-dimensionality; the continuation of the line of the lower leg into the interior of the foot shape gives the clue that the toe is farther from us than the heel. A *gesture* line is executed quickly, recording the path of the eye and depicting motion.

A mathematical curve can be thought of geometrically, as a locus of points, or algebraically, as a set of points satisfying an equation. Curves that model numerical data are usually algebraic so that we can make predictions from the equation. Geometric curves can also be considered to be modeling data—data that are points rather than numbers.

LESSON

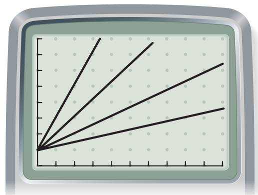
4.1

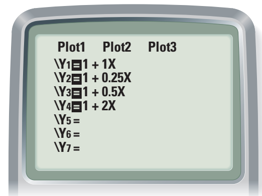
The nearest thing to nothing that anything can be and still be something is zero.

ANONYMOUS

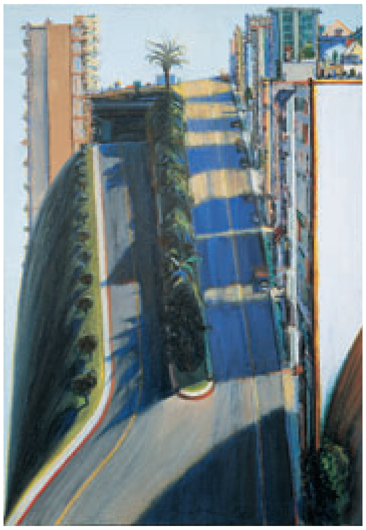
A Formula for Slope

You have seen that the steepness of a line can be a graphical representation of a real-world rate of change like a car's speed, the number of calories burned with exercise, or a constant relating two units of measure. Often you can estimate the rate of change of a linear relationship just by looking at a graph of the line. Can you tell which line in the graph matches which equation?

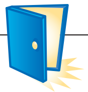




Slope is another word used to describe the steepness of a line or the rate of change of a linear relationship. In this investigation you will explore how to find the slope of a line using two points on the line.



Wayne Thiebaud's oil painting *Urban Downgrade, 20th and Noe* (1981) is an artistic representation of the steepness, or slope, of a street in San Francisco, California. Thiebaud is an American artist born in 1920.



Investigation
Points and Slope

You will need

- graph paper

Hector recently signed up with a limited-usage Internet provider. There is a flat monthly charge and an hourly rate for the number of hours he is connected during the month. The table shows the amount of time he spent using the Internet for the first three months and the total fee he was charged.

Internet Use

Month	Time (h)	Total fee ($)
September	40	16.55
October	50	19.45
November	80	28.15

Step 1 Is there a linear relationship between the time in hours that Hector uses the Internet and his total fee in dollars? If so, why do you think such a relationship exists? yes

Step 2 Use the numbers in the table to find the hourly rate in dollars per hour. Explain how you calculated this rate. $0.29/h

LESSON

4.1

PLANNING

LESSON OUTLINE

One day:

5 min	Introduction
20 min	Investigation
5 min	Sharing
10 min	Example and Visual Learning
5 min	Closing
5 min	Exercises

MATERIALS

- graph paper
- rulers
- The Four Slope Types (T), *optional*
- Coordinate Plane (T, from Chapter 1), *optional*
- Calculator Note 3D
- Sketchpad demonstration Slope, *optional*

TEACHING

The steepness of the graph of the line $y = a + bx$ is measured by the rate of change b, called the *slope* of the line.

INTRODUCTION

[ELL] A common meaning of *slope* is a piece of ground that isn't horizontal, such as a ski slope. In mathematics, slope is a number measuring steepness.

[Ask] "How are the lines on page 215 alike and how do they differ? How are these similarities and differences reflected in the equations?" [All the lines have a y-intercept of 1, but the slopes are different. The steepest line will match the equation with the largest rate of change. From bottom to top, the corresponding equations are Y2, Y3, Y1, and Y4.]

NCTM STANDARDS

CONTENT		PROCESS	
✓	Number	✓	Problem Solving
✓	Algebra	✓	Reasoning
	Geometry	✓	Communication
✓	Measurement		Connections
	Data/Probability	✓	Representation

LESSON OBJECTIVES

- Investigate and solve real-world problems that involve the slope of a line
- Learn how to calculate slopes with slope triangles and the slope formula
- Learn about slopes of rising, falling, horizontal, and vertical lines

Guiding the Investigation

One Step
Point out Hector's bill and ask students to write careful instructions for finding the slope of a line between any two given points. As students work, suggest that they use slope triangles.

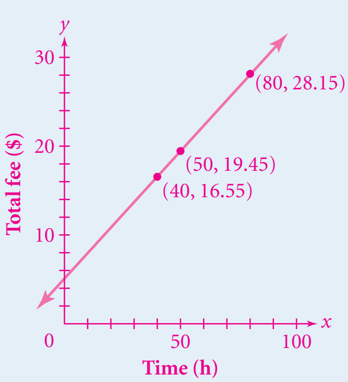
Step 1 Students can decide the relationship is linear either by graphing or by finding that the rate of change (in \$/h) is constant.

Step 2 **[Alert]** Some students may not realize that there is a monthly charge. This relationship is not a direct variation, and the intercept will not be zero.

Step 2 From September to October, the hours increased by 10 and the cost increased by \$2.90, so the increase is \$0.29/h. From October to November, the hours increased by 30 and the cost increased by \$8.70, also an increase of \$0.29/h.

Step 3 Ask students to label their axes with their respective units of measure as well as to mark them numerically.

Step 3 The line should support the linear relationship students assumed in Step 1.



Step 4 **[Alert]** Students may not understand how to find the lengths of the arrows. Suggest that they draw horizontal and vertical lines to mark equal lengths on the axes. Use the Coordinate Plane transparency to demonstrate this process.

Step 3 Draw a pair of coordinate axes on graph paper. Use the x-axis for time in hours and the y-axis for total fee in dollars. Plot and label the three points the table of data represents. Draw a line through the three points. Does this line support your answer in Step 1?

Step 4 Choose two points on your graph. Use arrows to show how you could move from one point to the other using only one vertical move and one horizontal move. How long is each arrow? What are the units of these values?

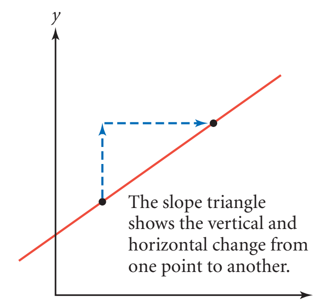
Step 5 The arrows show the changes in total fee and time usage, which are divided to find the rate. The slope is \$0.29/h, as in Step 2.

Step 5 How do the arrow lengths relate to the hourly rate that you found in Step 2? Use the arrow lengths to find the hourly rate of change, or slope, for this situation. What units should you apply to the number?

In Step 4, you used arrows to show the vertical change and the horizontal change when you moved from one point to another. The right triangle you created is called a **slope triangle.**

Step 6 Any pair of points will give the same slope.

Step 6 Choose a different pair of points on your graph. Create a slope triangle between them and use it to find the slope of the line. How does this slope compare to your answers in Step 2 and Step 5?



Step 7 Subtracting corresponding coordinates gives the arrows' lengths. Using (40, 16.55) and (80, 28.15), a numerical expression would be $\frac{28.15 - 16.55}{80 - 40}$.

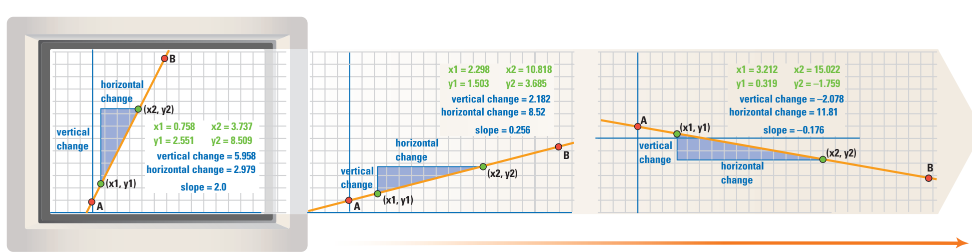
Step 7 Think about what you have done with your slope triangles. How could you use the coordinates of any two points to find the vertical change and the horizontal change of each arrow? Write a single numerical expression using the coordinates of two points to show how you can calculate slope.

Step 8 $\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{y_1 - y_2}{x_1 - x_2}$

Step 8 Write a symbolic algebraic rule for finding the slope between any two points (x_1, y_1) and (x_2, y_2). The subscripts mean that these are two distinct points of the form (x, y).

keymath.com/DA

[▶ Explore more about slope using the **Dynamic Algebra Exploration** at www.keymath.com/DA . ◀]



A slope triangle helps you visualize slope by showing you the vertical change and the horizontal change from one point to another. These changes are also called the "change in y" (vertical) and the "change in x" (horizontal). The example will help you see how to work with positive and negative numbers in slope calculations.

Students may draw the horizontal arrow before the vertical arrow, and the slope triangle may lie below the line.

Step 4 Arrows should form a right triangle. With the points (50, 19.45) and (80, 28.15) as an example, the horizontal arrow has length 30 h and the vertical arrow has length \$8.70.

Step 5 **[Alert]** Some students may divide the horizontal change by the vertical change. As a guide, have them think about the units of the rate: \$/h, not h/\$.

Step 7 Students revisit what may have caused confusion in Step 4. When you think someone might not understand deeply enough, ask for explanations.

Step 8 This may be the first time students have seen subscripted variables. You might mention that x_1 is read "x-sub-one."

EXAMPLE

Consider the line through the points (1, 7) and (6, 4).

a. Find the slope of the line.

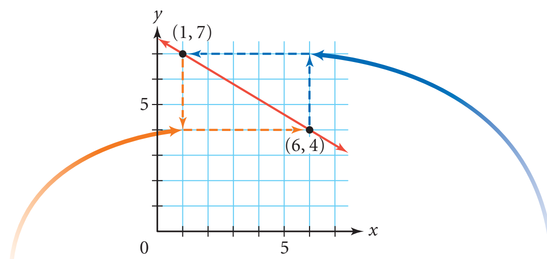
b. Without graphing, verify that the point (4, 5.2) is also on that line.

c. Find the coordinates of another point on the same line.

▸ *Solution*

Plot the given points and draw the line between them.

a. There are two different slope triangles you could draw using these points.



If you move from (1, 7) to (6, 4), the change in y is -3 (down 3) and the change in x is $+5$ (right 5). The slope is $\frac{-3}{+5}$.

If you move from (6, 4) to (1, 7), the change in y is $+3$ (up 3) and the change in x is -5 (left 5). The slope is $\frac{+3}{-5}$.

$\frac{-3}{+5}$ is equivalent to $\frac{+3}{-5}$. You get the same slope, $-\frac{3}{5}$ or -0.6, no matter which point you start from. The slope triangles help you see this relationship more clearly.

Move to (6, 4) from (1, 7).

$$\text{Slope} = \frac{4-7}{6-1} = \frac{-3}{5} = -\frac{3}{5} \quad \text{or}$$

Move to (1, 7) from (6, 4).

$$\text{Slope} = \frac{7-4}{1-6} = \frac{3}{-5} = -\frac{3}{5}$$

b. The slope between any two points on the line will be the same. (And, the slope between a point on the line and a point not on the line will be different.) So, if the slope between the point (4, 5.2) and either of the original two points is -0.6, then the point is on the line. The slope between (4, 5.2) and (1, 7) is

$$\frac{7-5.2}{1-4} = \frac{1.8}{-3} = -\frac{1.8}{3} = -0.6$$

So the point (4, 5.2) is on the line.

To find the slope when moving from (1, 7) to (6, 4), the coordinates of (1, 7) will come last in the slope formula. Saying to move to (6, 4) from (1, 7) mentions the points in the order they appear in the slope formula.

SHARING IDEAS

Request that students present Steps 5 and 8. Ask why the ratio of differences gives the slope. Keep asking "Are you saying . . . ?" to clarify thinking and help all students understand these important but difficult ideas.

[Ask] "Is a linear equation a good model?" [The relationship applies only to whole number values of x so the points on the line between the whole numbers have no meaning.]

Mention various descriptions of slope. For example, "rise over run," "vertical change over horizontal change," and "the change in y over the change in x." Draw out the formula (as written at the bottom of page 218), and write the slope using deltas: $\frac{\Delta y}{\Delta x}$. Delta is the Greek letter "D," standing here for "difference." **[Language]** A *delta* also is defined as a triangular deposit built up at the mouth of a river.

If students worked on the project Legal Limits from Lesson 3.5, point out that slope is the same as gradient. Slope is usually given as a fraction and gradient as a percent. A 9% gradient is a slope of $\frac{9}{100}$.

[Ask] "If unlimited service costs $21.95 per month, do you think Hector should change pricing plans or stay with limited usage?"

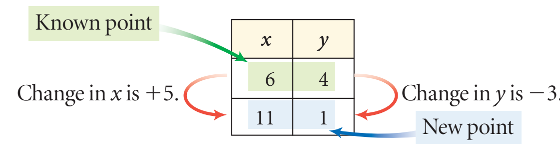
Assessing Progress
Note students' understanding of linear relationships and their ability to find rates and to graph points and to treat horizontal and vertical components separately.

EXAMPLE

This example provides a method for finding the slope of a line between two points. It also introduces the notion that a point C is on the same line as points A and B if and only if the slope of the segment AC is the same as the slope of the segment AB.

[Ask] "If you know the equation for the line—say, $y = 7.6 - 0.6x$—how might you find another point on the line?" Encourage a variety of approaches: using the slope and (0, 7.6), substituting a number for x and determining y, graphing, and so on. Have students work on the last two questions in part c for a few minutes to get $(1 - 5, 7 + 3) = (-4, 10)$ and either $(1 + 1, 7 - 0.6) = (2, 6.4)$ or $(6 + 1, 4 - 0.6) = (7, 3.4)$.

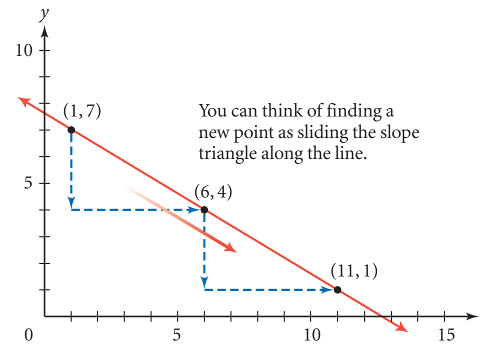
c. You can find the coordinates of another point by adding the change in x and the change in y from any slope triangle on the line to a known point.



Starting with the point (6, 4) and using

$$\frac{\textit{change in } y}{\textit{change in } x} = \frac{-3}{5}$$

gives the new point $(6 + 5, 4 + (-3)) = (11, 1)$.



Try using the point (1, 7) and using

$$\frac{\textit{change in } y}{\textit{change in } x} = \frac{3}{-5} \quad (-4, 10)$$

to find another point. Try using either original point and using

$$\frac{\textit{change in } y}{\textit{change in } x} = \frac{-0.6}{1} \quad (7, 3.4) \text{ or } (2, 6.4)$$

to find another point.

Slope is an extremely important concept in mathematics and in applications like medicine and engineering that rely on mathematics. You may encounter different ways of describing slope—for example, "rise over run" or "vertical change over horizontal change." But you can always calculate the slope using this formula:

History CONNECTION

Slope is sometimes written $\frac{\Delta y}{\Delta x}$. The symbol Δ is the Greek capital letter delta. The use of Δ is linked to the history of calculus in the 18th century when it was used to mean "difference."

Slope Formula

The formula for the **slope** of the line passing through point 1 with coordinates (x_1, y_1) and point 2 with coordinates (x_2, y_2) is

$$\text{slope} = \frac{\textit{change in } y}{\textit{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

VISUAL LEARNING

As needed, review division with signed numbers. Students may forget, for example, that the quotient of two negative numbers is a positive number.

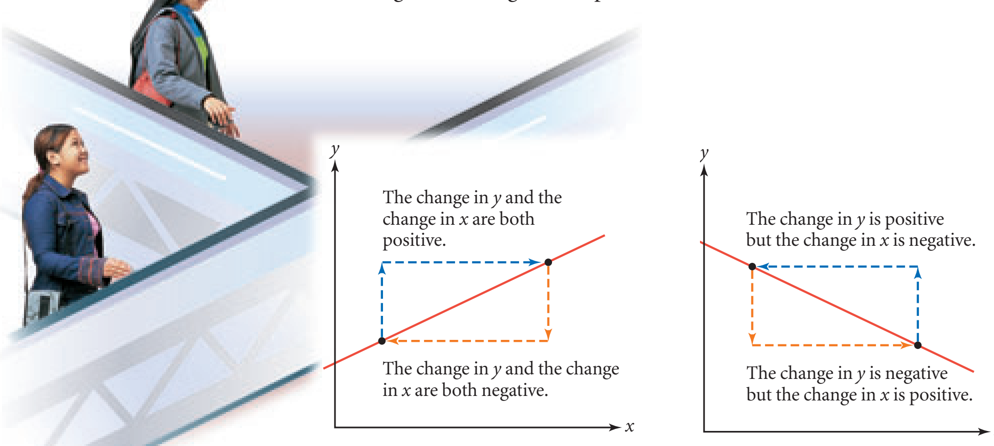
Emphasize that either point can be called (x_1, y_1), as long as subtraction between corresponding coordinates takes place in the same order in the numerator and the denominator.

Visual learners will also be helped by a graphic display such as this:

	x	y	
$x_2 - x_1$	x_1	y_1	$y_2 - y_1$
	x_2	y_2	

Be sure all students now recognize that a is the intercept and b is the slope in the equation $y = a + bx$.

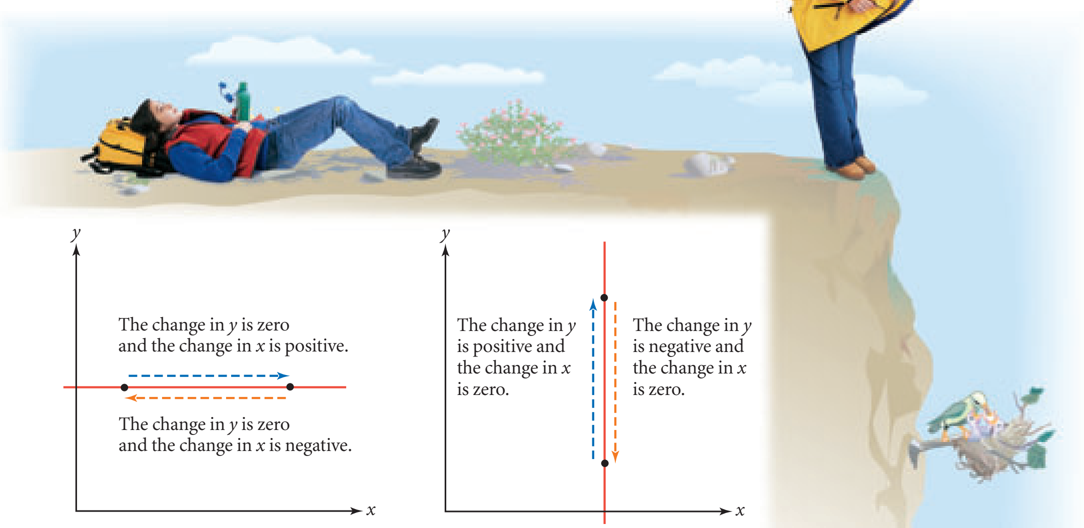
A line that goes up from left to right has a positive slope. A line that goes down from left to right has a negative slope.



The ratio $\frac{\textit{change in } y}{\textit{change in } x}$ is positive, so the slope is positive.

The ratio $\frac{\textit{change in } y}{\textit{change in } x}$ is negative, so the slope is negative.

Horizontal lines have a slope of zero because they have no change in y. Vertical lines have no change in x. To calculate the slope of a vertical line, you would have to divide by zero, which is impossible—we say that the slope of a vertical line is undefined.



The ratio becomes $\frac{0}{\textit{change in } x}$, so the slope is zero.

The ratio becomes $\frac{\textit{change in } y}{0}$, so the slope is undefined.

As you work on the exercises, keep in mind that the slope of a line is the same as the rate of change of its equation. When a linear equation is written in intercept form, $y = a + bx$, which letter represents the slope?

Dynamic Concept of Slope

Imagine that you're moving from left to right along the line. What happens to your y-coordinate? If your y-coordinate is increasing, the slope is positive. You're climbing up the line. The larger the slope, the harder you have to work to make that climb. If your y-coordinate is decreasing, you're going downhill. The more negative the rate of change, the steeper the hill.

The dynamic approach can help students understand slopes of horizontal and vertical lines, which can be especially confusing. If your y-coordinate doesn't change, you're moving "on the level," so the line is horizontal. Its slope is zero. If, on the other hand, your motion can't be from left to right at all because the line is vertical, it's impossibly steep. The slope is undefined. If you calculate the slope of a vertical line by the static model, you're dividing by zero, the difference in x-coordinates of two points on the line.

Dividing by Zero

If students ask why it is impossible to divide by 0, you might say, "If it were possible to divide 5 by 0, then the answer multiplied by 0 would equal 5." Or you might ask, "What number multiplied by 0 would equal 5?" [No such number exists, so $\frac{5}{0}$ is undefined.] If a student asks about the quotient $\frac{0}{0}$, say that $\frac{179}{541}$ (or any other number) multiplied by 0 equals 0. Any number could be the answer, so $\frac{0}{0}$ isn't defined.

Closing the Lesson

As needed, say that the **slope** of a line can be calculated by finding two points on the line and dividing the vertical change between those points by the corresponding horizontal change. You can use the Four Slope Types transparency.

You can use the Sketchpad demonstration Slope to further explore slope and to review the intercept form of linear equations.

BUILDING UNDERSTANDING

Students work with slopes to graph lines and to determine which points lie on given lines. They also relate slopes to rates of change in real-world problems.

ASSIGNING HOMEWORK

Essential	1–6, 8, 11
Performance assessment	10
Portfolio	7
Journal	9, 11c
Group	10
Review	12–16

Helping with the Exercises

2a. $\frac{3}{2}$, or 1.5; one possible point is (6, 10).

2b. $-\frac{3}{2}$, or -1.5; one possible point is (4, 2).

2c. 0; any point with a y-coordinate of 4 will be on the line.

2d. $\frac{15}{8}$, or 1.875; one possible point is (17, 27).

Exercise 3 This exercise stresses that the slope of a line is constant and can be calculated from any two points. You might ask students to use slope triangles rather than memorize the formula.

Exercise 4 This program is a fun way for students to gain experience in identifying the equations of lines.

Exercise 5 Be prepared to point out that the x-coordinates don't change on a vertical line. Students will see equations of horizontal and vertical lines again in Lesson 4.2.

5a. i. The x-values don't change, so the slope is undefined.

5a. ii. The y-values decrease as the x-values increase, so the slope is negative.

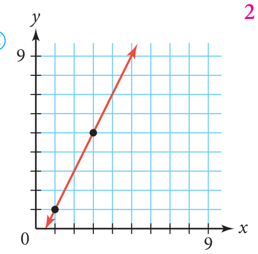
EXERCISES

You will need your graphing calculator for Exercises **4** and **14.**

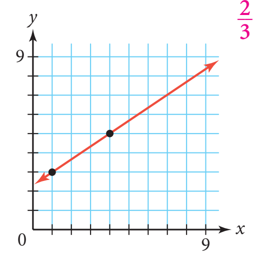
Practice Your Skills

1. Find the slope of each line using a slope triangle or the slope formula.

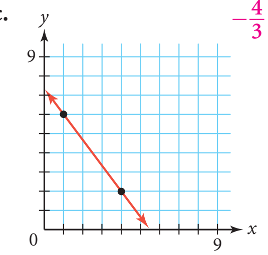
a. @ 2



b. $\frac{2}{3}$



c. $-\frac{4}{3}$



2. Find the slope of the line through each pair of points. Then name another point on the same line.

a. (2, 4), (4, 7) @

b. (6, -1), (2, 5)

c.

x	y
-2	4
8	4

d.

x	y
1	-3
9	12

3. Given the slope of a line and one point on the line, name two other points on the same line. Then use the slope formula to check that the slope between each of the two new points and the given point is the same as the given slope. Possible answers:

a. Slope $\frac{3}{1}$; point (0, 4) @ (1, 7), (-1, 1)

b. Slope -5; point (2, 8) (3, 3), (1, 13)

c. Slope $-\frac{3}{4}$; point (8, 6) (12, 3), (4, 9)

d. Slope 0.2; point (5, 7) (6, 7.2), (4, 6.8)

4. Run the LINES program five times. Start by playing the easy level once or twice, then move on to the difficult level. On your paper, sketch a graph of each randomly generated line. Find the slope of the line by counting the change in y and the change in x on the grid, or trace the line for two points to use in the slope formula. Then find the y-intercept and write the equation of the line in intercept form.
[▶ See **Calculator Note 3D** to learn how to use the LINES program. ◀] Answers will vary.

Reason and Apply

5. Each table gives the coordinates of four points on a different line.

i. @

x	y
4	-8
4	0
4	3
4	20

ii.

x	y
0	5
1	3
3	-1
4	-3

iii.

x	y
-4	-5
-3	-5
1	-5
4	-5

iv.

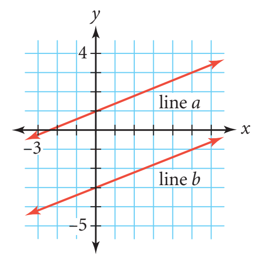
x	y
-4	-5
-2	-3.5
0	-2
4	1

a. Without calculating, can you tell whether the slope of the line through each set of points is positive, negative, zero, or undefined? If so, explain how you can tell.

b. Choose two points from each table and calculate the slope. Check that your answer is correct by calculating the slope with a different pair of points.

c. Write an equation for each table of values.

6. Consider lines a and b shown in the graph at right.



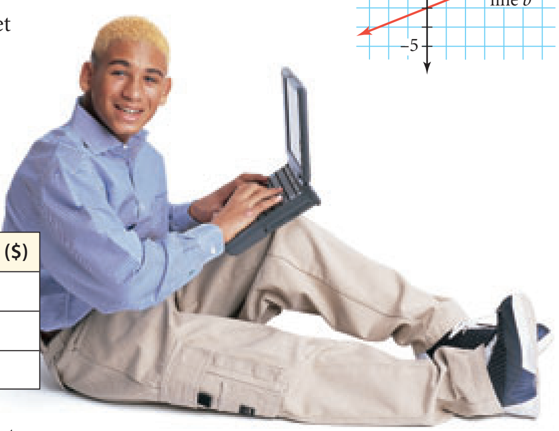
a. How are the lines in the graph alike? How are they different?

b. Which line matches the equation $y = -3 + \frac{2}{5}x$? line b

c. What is the equation of the other line? $y = 1 + \frac{2}{5}x$

d. How are the equations alike? How are they different?

7. **APPLICATION** Recall Hector's Internet use from the investigation. You probably found that his provider charges \$0.29 per hour of use—that was the slope of the line you graphed.



Internet Use

Month	Time (h)	Total fee ($)
September	40	16.55
October	50	19.45
November	80	28.15

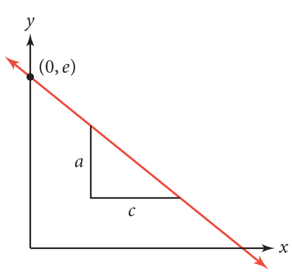
a. Use the rate of change and the data in the table to find out how much the total fee is for 30 h of use. How much is the total fee for 20 h? ⓐ

b. Repeat the process in 7a to find out how much the total fee is for 0 h of use. What is the real-world meaning of this number in this situation? (Look back at the investigation for help.) ⓐ

c. A mathematical model can be an equation, a graph, or a drawing that helps you better understand a real-world situation. Write a linear equation in intercept form that you can use to model this situation.

d. Use your linear equation to find out how much the total fee is for 280 h of use.
Substitute 280 for x and solve for y: $y = 4.95 + 0.29(280) = 86.15$, or \$86.15 for 280 h.

8. If a and c are the lengths of the vertical and horizontal segments and $(0, e)$ is the y-intercept, what is the equation of the line? ⓗ $y = e - \frac{a}{c}x$



5a. iii. The y-values don't change, so the slope is zero.

5a. iv. The y-values increase as the x-values increase, so the slope is positive.

5b. i. Using the points (4, 0) and (4, 3), the slope is $\frac{3-0}{4-4} = \frac{3}{0}$. You can't divide by 0, so the slope is undefined.

5b. ii. Using the points (1, 3) and (4, −3), the slope is $\frac{-3-3}{4-1} = -2$.

5b. iii. Using the points (−4, −5) and (−3, −5), the slope is $\frac{-5-(-5)}{-3-(-4)} = 0$.

5b. iv. Using the points (0, −2) and (4, 1), the slope is $\frac{1-(-2)}{4-0} = \frac{3}{4}$.

5c. i. $x = 4$; **ii.** $y = 5 - 2x$; **iii.** $y = -5$; **iv.** $y = -2 + \frac{3}{4}x$

Exercise 6 Students will often use the fact that lines are parallel when they have the same slope.

6a. The lines are parallel, so they have the same slope; the y-intercepts are different.

6d. The slope, $\frac{2}{5}$, is the same in each equation; the y-intercepts, −3 and 1, are different.

7a. Use the slope to move backward from (40, 16.55): $(40 - 10, 16.55 - 0.29 \cdot 10) = (30, 13.65)$, or \$13.65 for 30 h; $(30 - 10, 13.75 - 0.29 \cdot 10) = (20, 10.75)$, or \$10.75 for 20 h.

Exercise 7b This approach to finding the y-intercept by working backward is referred to in Lesson 4.3.

7b. Continuing the process in 7a leads to (0, 4.95), or \$4.95 for 0 h. This is the flat monthly rate for Hector's Internet service.

Exercise 7c Mathematical models have limitations. For example, the line contains points with negative x-values, but they don't make sense in the situation being modeled.

7c. $y = 4.95 + 0.29x$, where x is time in hours and y is total fee in dollars

Exercise 8 Some students may try measuring to find a numerical answer. Point out that when distances are represented by letters, students should express their answer in terms of those letters. **[ELL]** *In terms of* means *using*. The slope can't be written using only those letters, however. A minus sign is needed because the slope is negative. Stress again that a negative slope means the line is falling *from left to right*. Any of the expressions $y = e - \frac{a}{c}x$, $y = e + \frac{-a}{c}x$, or $y = e + \frac{a}{-c}x$ are okay.

9a. The change in y and the change in x are the same for any slope triangle on the line.

9b. A steeper line would have a greater change in y than its change in x. Numerically, the slope would be greater than 1.

9c. A less steep line would have a greater change in x than its change in y. Numerically, the slope would be between 0 and 1.

9d. The line would go down from left to right, because the slope is negative; the line would be very steep, because 15 is significantly greater than 1.

Exercise 10 Some students may be confused about how the height of the balloon can already be 14 m at time 0 min. If possible, ask the class to explain it. There can be several reasons. The timing need not start when the balloon starts rising. Or the measurement might be to the top of the balloon, which might very well be 14 m high when the basket is still on the ground. In case the balloon has not started rising, the time-distance relationship would not be perfectly linear. To explain, you might note that when you start walking, you have some acceleration before you obtain any speed. Hence, the beginning of the graph would be slightly curved.

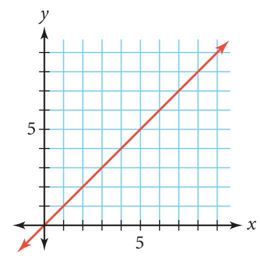
[Ask] "How can you tell whether the data are linear?" [Slope between pairs of points is constant.]

10b. m/min; the hot-air balloon rises at a rate of 30 m/min.

10c. $y = 14 + 30x$

Exercise 11 This is a good exercise for leading up to Lesson 4.2.

9. This line has a slope of 1. Graph it on your own paper.



a. Draw a slope triangle on your line. How do the change in y and the change in x compare?

b. Draw a line that is steeper than the given line. How do the change in y and the change in x compare? How does the numerical slope compare to that of the original line?

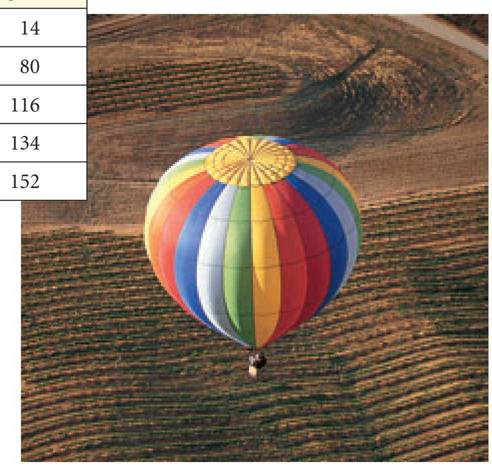
c. Draw a line that is less steep than the given line, but still increasing. How do the change in y and the change in x compare? How does the numerical slope compare to that of the given line?

d. How would a line with a slope of -15 compare to your other lines? Explain your reasoning.

10. APPLICATION A hot-air balloonist gathered the data in this table.

Hot-Air Balloon Height

Time (min)	Height (m)
0	14
2.2	80
3.4	116
4	134
4.6	152



a. What is the slope of the line through these points? 30 m/min

b. What are the units of the slope? What is the real-world meaning of the slope? @

c. Write a linear equation in intercept form to model this situation.

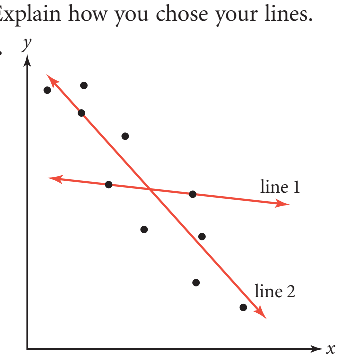
d. What is the height of the balloon after 8 min? @ 254 m

e. During what time interval is the height less than or equal to 500 m? between 0 and 16.2 min

11. When you make a scatter plot of real-world data, you may see a linear pattern.

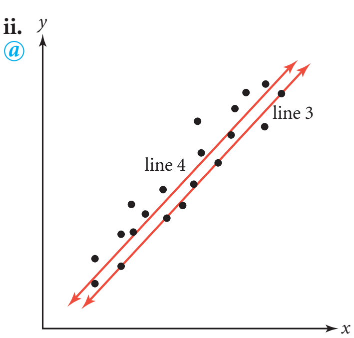
a. Which line do you think "fits" each scatter plot? Think about slope and how the points are scattered. Explain how you chose your lines.

i.



Line 2 is a better choice. A majority of points are closer to line 2 than to line 1.

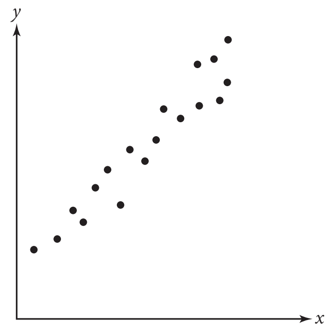
ii. @



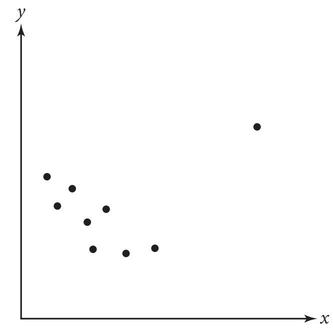
Line 4 is a better choice. Line 3 passes through or is close to a good number of points, but too many points are above this line and too few are below it. Even though line 4 does not intercept any points, it is the better choice because about the same number of points are above the line as below it.

b. Trace each scatter plot onto your own paper. Then draw a line that you think fits the data.

i.



ii.



c. List two features that you think are important for a line that fits data.

Review

1.7 **12.** The base of a triangle was recorded as 18.3 ± 0.1 cm and the height was recorded as 7.4 ± 0.1 cm. These measurements indicate the measured value and an accuracy component.

a. Use the formula $A = 0.5bh$ and the measured values for base and height to calculate the area of the triangle. $0.5(18.3)(7.4) = 67.7\text{ cm}^2$

b. Use the smallest possible lengths for base and height to calculate an area. ⓐ $0.5(18.2)(7.3) = 66.4\text{ cm}^2$

c. Use the largest possible lengths for base and height to calculate an area. $0.5(18.4)(7.5) = 69.0\text{ cm}^2$

d. Use your answers to 12a–c to express the range of possible area values as a number ± an accuracy component. ⓗ $67.7 \pm 1.3\text{ cm}^2$

1.2 **13.** Calista has five brothers. The mean of her brothers' ages is 10 years, and the median is 6 years. Create a data set that could represent the brothers' ages. Is this the only possible answer? {3, 3, 6, 16, 22}; no

2.8 **14.** Enter {−3, −1, 2, 8, 10} into list L_1 on your calculator.

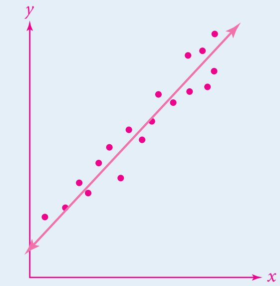
a. Write a rule for list L_2 that adds 14 to each value in list L_1 and then multiplies the results by 2.5. What are the values in list L_2? ⓐ

b. Write a rule for list L_3 that works backward and undoes the operations in list L_2 to produce the values in list L_1. ⓐ $L_3 = \frac{(L_2 - 35)}{2.5}$, or $L_3 = \frac{L_2}{2.5} - 14$

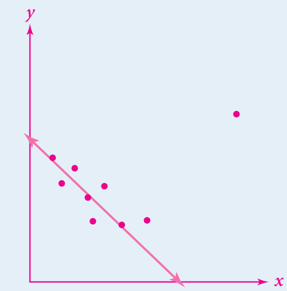
2.2 **15.** Convert each decimal number to a percent.

a. 0.85 85% **b.** 1.50 150% **c.** 0.065 6.5% **d.** 1.07 107%

11b. i.



11b. ii.



11c. Answers will vary. The line should reflect the direction of the data, and about the same number of points should be above the line as below it.

Exercise 12 [Alert] Students may calculate to two decimal places, finding that the minimum is 67.71 − 1.28 and the maximum is 67.71 + 1.29. The initial measurements were to one decimal place, so these numbers must also be to one decimal place.

14a. $L_2 = 2.5(L_1 + 14)$; {27.5, 32.5, 40, 55, 60}

3.6 **16.** The equation $7x - 10 = 2x + 3$ is solved by balancing. Explain what happens in Stages 3, 4, and 6 of the balancing process.

$2x - 10 = 7x + 3$	1. Original equation.
$2x - 2x - 10 = 7x - 2x + 3$	2. Subtract $2x$ from both sides.
$-10 = 5x + 3$	3. Combine like terms.
$-10 - 3 = 5x + 3 - 3$	4. Subtract 3 from both sides.
$-13 = 5x$	5. Combine like terms.
$\frac{-13}{5} = \frac{5x}{5}$	6. Divide both sides by 5.
$x = -2.6$	7. Reduce.
$2(-2.6) - 10 \stackrel{?}{=} 7(-2.6) + 3$	
$-5.2 - 10 \stackrel{?}{=} -18.2 + 3$	
$-15.2 = -15.2$	Solution checks.

project

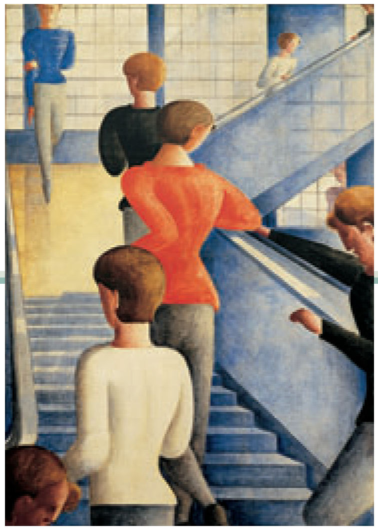
STEP RIGHT UP

How would it feel to climb a flight of stairs if every step were a little taller or shorter, or wider or narrower, than the previous one? The constant measure for treads and risers on most stairs keeps you from tripping. Have you noticed that the stairs outside some public buildings slow you down to a "ceremonial" pace? Or that little-used stairs to a cellar seem dangerously steep? Investigate the standards for stairs in various architectural settings and learn the reasons for their various slopes.

Your project should include

- Tread-and-riser data and slope calculations for several different stairways.
- The building codes or recommended standards in your area for home stairways. Is a range of slopes permitted? When are landings or railings required?
- Scale drawings for at least three different stairways.

After you've done your research, consider this question: Does a spiral staircase have a constant slope?



Slope triangles are like the steps of a staircase. This oil painting, *Bauhaus Stairway*, by the German artist Oskar Schlemmer (1888–1943) shows many slope triangles.

Bauhaus Stairway (1932). Oil on canvas, 63-7/8 × 45 in. The Museum of Modern Art, New York. Gift of Philip Johnson. Photograph © 2000 The Museum of Modern Art, New York

STEP RIGHT UP PROJECT

[ELL] The *tread* of a stair step is the horizontal part. The *riser* is the vertical piece connecting treads.

General information about treads and risers is available in home-improvement books.

Supporting the project

MOTIVATION

If you were going to design a stairway for a child's play structure, would you use the same standards as for an adult house? If not, what would be the difference and why?

OUTCOMES

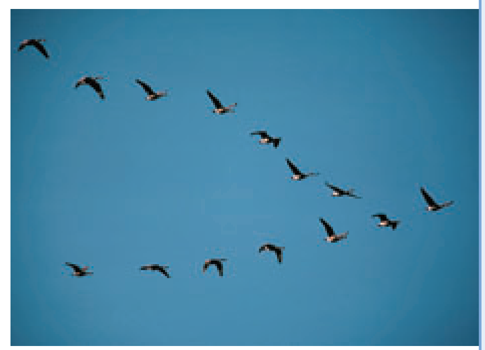
- The report is accurate on all required elements.
- The report on building codes cites sources.
- The report includes spiral staircases. To determine their slope, take measurements at the same distance from the center of the circle the staircase is based on.
- The scale for scale drawings of different stairways is marked.
- The report discusses safety.
- The paper lists reasonable factors for determining the slope within the allowable range, such as frequency of use or user age and agility.

LESSON

4.2

Writing a Linear Equation to Fit Data

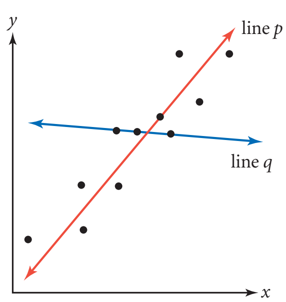
When you plot real-world data, you will often see a linear pattern. If you can find a line or an equation to model linear data, you can make predictions about unknown data values. However, data points rarely fall exactly on a line. How can you tell if a particular line is a good model for the data? One of the simplest ways is to ask yourself if the line shows the general direction of the data and if there are about the same number of points above the line as below the line. If so, then the line will appear to "fit" the data, and we call it a **line of fit.**



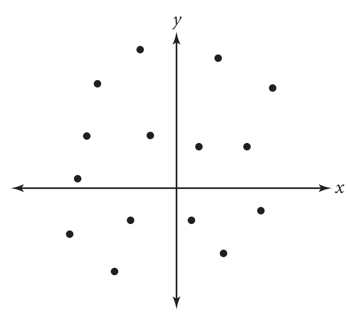
Can you visualize two lines that model this arrangement of geese?

Sometimes one line will be a better model for your data than another. Each of these graphs shows a scatter plot of data points and possible lines of fit.

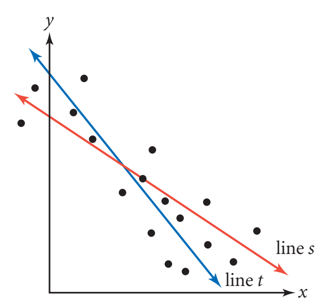
Graph A



Graph B



Graph C



In Graph A, line p fits better because it shows the general direction of the data and there are the same number of points above the line as below the line. Although line q goes through several points, it does not show the direction of the data.

In Graph B, the data don't seem to have a pattern. No lines of fit are shown because you can't say that one line would fit the data better than another line would.

In Graph C, both lines show the general direction of the data and both lines have the same number of points above and below them. You could consider either line a line of fit. When making predictions, how would your calculations using the equation for line s differ from those using the equation for line t?

In the next investigation you will learn one method to find a possible line of fit.

NCTM STANDARDS

CONTENT		PROCESS	
✓	Number	✓	Problem Solving
✓	Algebra	✓	Reasoning
	Geometry	✓	Communication
	Measurement	✓	Connections
✓	Data/Probability	✓	Representation

LESSON OBJECTIVES

- Draw a line that fits or models a set of points
- Write an intercept equation that fits a set of real-world data

LESSON

4.2

PLANNING

LESSON OUTLINE

First day:

15 min Introduction

35 min Investigation (Steps 1–8)

Second day:

10 min Investigation (Steps 9–12)

10 min Sharing

20 min Example

5 min Closing

5 min Exercises

MATERIALS

- graph paper (several sheets per group)
- box of (uncooked) spaghetti, 10 in. long
- several books for each group
- 5 oz plastic cups (one per group)
- string
- pennies (100 per group)
- empty boxes (one per group), *optional*
- sharp scissors or awls (to punch cups for string)
- Fathom demonstration Fast Food, *optional*

TEACHING

A *line of fit* models a data set to allow predictions. In this lesson students work with slopes and intercepts to gain an intuitive feeling for finding lines of fit, otherwise known as *trend lines.*

Guiding the Investigation

Procedure Note You might use angel hair pasta so that fewer pennies are needed to break a beam. Using similar books allows students to make stacks of the same height. Facilitate cleanup by putting a box under each experiment to catch the broken spaghetti. To save time, you can divide the Step 3 tasks between groups, and create one data set for the class.

One Step
Ask students to experiment with 1 to 6 strands of spaghetti and predict how many pennies 17 strands of spaghetti will support and how many spaghetti strands will be needed to support $5 worth of pennies. As you circulate, suggest that students lay out spaghetti strands on the graphs they create to help them visualize lines of fit.

Step 1 The string should be tied short to keep the cup off the base.

Step 2 **[Language]** The *maximum load* is the number of pennies in the cup *before* the one is added that breaks the beam.

As students devise their own tables for recording data, suggest that they think about the input and output variables in deciding how to title the columns.

Step 3 You may want to ask, "Would your results be the same if you moved the books closer or farther apart?" "Are your results with spread-out spaghetti strands different from those with bunched-up strands?"

Steps 4–7 These steps are to be worked on individually. Results will be shared with the group in Step 8.

Students might also make an accurate sketch on graph paper and then use a strand of spaghetti to visualize the line of fit. To get to an equation in intercept form, they might draw a slope triangle for the spaghetti and approximate where the spaghetti crosses the y-axis.

Investigation
Beam Strength

You will need

- graph paper
- uncooked spaghetti
- several books
- a plastic cup
- string
- pennies

How strong do the beams in a ceiling have to be? How do bridge engineers select beams to support traffic? In this investigation you will collect data and find a linear model to determine the strength of various "beams" made of spaghetti.

Steps 1–7 Answers will vary.

Procedure Note

1. Hang your cup at the center of your spaghetti beam.
2. Support the beam between the stacks of books so that it overlaps each stack by about 1 inch. Put another book on each stack to hold the beam in place.
3. Put pennies in the cup, one at a time, until the beam breaks.

Step 1 Make two stacks of books of equal height. Punch holes on opposite sides of the cup and tie the string through the holes.

Step 2 Follow the Procedure Note for a beam made from one strand of spaghetti. Record the maximum load (the number of pennies) that this beam will support.

Step 3 Repeat Step 2 for beams made from two, three, four, five, and six strands of spaghetti.

Step 4 Plot your data on your calculator. Let x represent the number of strands of spaghetti, and let y represent the maximum load. Sketch the plot on paper too.

Step 5 Use a strand of spaghetti to visualize a line that you think fits the data on your sketch. Choose two points on the line. Note the coordinates of these points. Calculate the slope of the line between the two points.

Step 6 Use the slope, b, that you found in Step 5 to graph the equation $y = bx$ on your calculator. Why is this line parallel to the direction the points indicate? Is the line too low or too high to fit the data?

Step 7 Using the spaghetti strand on your sketch, estimate a good y-intercept, a, so that the equation $y = a + bx$ better fits your data. On your calculator, graph the equation $y = a + bx$ in place of $y = bx$. Adjust your estimate for a until you have a line of fit.

Step 8 Students should see that everyone could come up with a different line of fit. Some lines may pass through data points, but it is not a requirement.

Step 8 In Step 5, everyone started with a visual model that went through two points. In your group, compare all final lines. Did everyone end up with the same line? Do you think a line of fit must go through at least two data points? Is any one line better than the others?

Step 4 In making a scatter plot on a calculator, using a box or a plus mark makes it easier to see a line of fit than using a dot mark. You might encourage students to use list L1 for input data and list L2 for output data whenever possible, to avoid having to specify lists each time they make a plot.

Step 7 You may want to write the equation $y = bx$ as $y = 0 + bx$ to help students see the connection to $y = a + bx$ and the y-intercept of zero. As needed, remind them of the meaning of *y-intercept.*

Your line is a model for the relationship between the number of strands of spaghetti in the beam and the load in pennies that the beam can support.

Step 9 the additional number of pennies each additional strand of spaghetti can hold

Step 9 Explain the real-world meaning of the slope of your line.

Step 10 Use your linear model to predict the number of spaghetti strands needed to support $5 worth of pennies.

Step 11 Use your model to predict the maximum loads for beams made of 10 and 17 strands of spaghetti.

Step 12 errors in data collection or a poor line of fit

Step 12 Some of your data points may be very close to your line, while others could be described as outliers. What could have caused these outliers?

Despite engineering tests, buildings can suffer damage during stress. This building in San Francisco, California, collapsed during an earthquake in October 1989.

Engineers conduct tests using procedures similar to the one you used in your investigation. The test results help them select the best materials and sizes for beams in buildings, bridges, and other forms of architecture.

EXAMPLE

This table shows how many fat grams there are in some hamburgers sold by national chain restaurants.

This is a ceramic sculpture of a hamburger. Imagine how much total fat this burger would have if it were real!

Hamburger (1983) by David Gilhooly, Collection of Harry W. and Mary Margaret Anderson, Photo by M. Lee Fatheree

Nutrition Facts

Burger	Saturated fat (g)	Total fat (g)
Burger King Bacon Double Cheeseburger	17	34
Burger King Original WHOPPER® Sandwich with Cheese	18	49
Hardee's 2/3 lb Double Thickburger	38	90
Hardee's 2/3 lb Bacon Cheese Thickburger	40	96
Jack in the Box Bacon Ultimate Cheeseburger	29	70.5
Jack in the Box Jumbo Jack with Cheese	16	41.5
McDonald's Big Mac	11	33
McDonald's Quarter Pounder	8	21
Wendy's Jr. Hamburger	3.5	9
Wendy's Classic Single with Everything	7	19

(*www.burgerking.com, www.hardeesrestaurants.com, www.jackinthebox.com, www.mcdonalds.com, www.wendys.com*)

a. Find a linear equation to model the data (*saturated fat, total fat*).

b. Tell the real-world meanings of the slope and intercept of your line.

c. Predict the total fat in a burger with 20 g of saturated fat.

d. Predict the saturated fat in a burger with 50 g of total fat.

SHARING IDEAS

[Language] You may want to introduce the term *extrapolate*, meaning to estimate outside the observed range.

Pick students who will present a variety of answers to Steps 10 and 11. (If students are putting their scatter plots on a calculator with an overhead projection panel, placing a spaghetti strand on the panel is good for showing a line of fit.) Point out that lines of fit that are only slightly different on a graph can produce drastically different results when extended very far.

Ask the class to decide which results they find most believable and why. Motivate the need for an objective, standard way to find a line of best fit, foreshadowing Lesson 4.6.

If the opportunity arises, you might ask how to get the equation by shifting the line through the origin, not just up but also to the right, to go through one of the two points chosen. This could motivate Lesson 4.3.

You might say that a mathematical model or a line of fit is an idealization. Rarely will the data fit the model exactly. In fact, in the case of beams, the strength is proportional to the cross-sectional area (and thus to the square of the number of spaghetti strands) and inversely proportional to the length of the beam between supports. So a linear model isn't a very good predictor beyond a few strands.

Assessing Progress
Watch for students' understanding of input-output tables and their skill at making scatter plots, collecting data systematically, working with a group, finding the slope and y-intercept of a line, writing the intercept form of the equation of a line, evaluating an equation at a point, and solving a linear equation.

EXAMPLE

This example is for students who had difficulty with the investigation. You can use the Fathom demonstration Fast Food to replace this example.

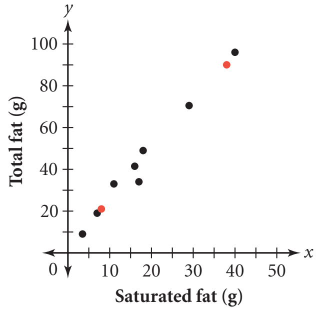
In the solution to part a, students may ask how to determine how far the line should be raised. Point out that they can use the graph to guess at the y-intercept. The line $y = 2.6 + 2.3x$ goes through the points used to find the slope, but that line has more data points above the line than below.

MAKING THE CONNECTION

Saturated and trans fats increase the amount of cholesterol in the blood, leading to increased risk of heart disease and hardening of the arteries. Unsaturated fat doesn't increase cholesterol.

▶ Solution

Draw a scatter plot of the data. Let x be the number of grams of saturated fat, and let y be the total number of grams of fat.



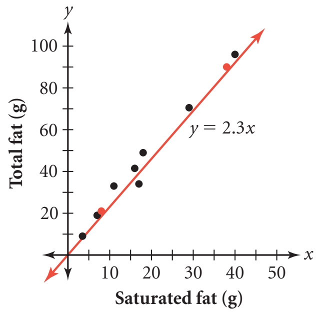
a. The scatter plot shows a linear pattern in the data. A line through the points (8, 21) and (38, 90) seems to show the direction of the data. Calculate the slope b of the line between these two points.

$$b = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90 - 21}{38 - 8} = \frac{69}{30} = 2.3$$

Substitute 2.3 for b in $y = bx$ to get

$$y = 2.3x$$

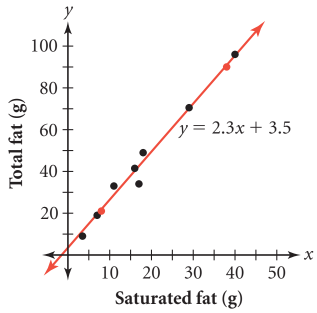
The equation $y = 2.3x$ shows the direction of the line, but has only one point below the line and the other nine above.



Adjust the y-intercept by tenths until you find a line that appears to be a good fit for the data. You may find that the equation

$$y = 3.5 + 2.3x$$

is a good model. Notice that the line of fit doesn't have to go through any data points.



b. The y-intercept, 3.5, means that even without any saturated fat, a burger has about 3.5 grams of total fat. The slope, 2.3, means that for each additional gram of saturated fat there are an additional 2.3 grams of total fat.

c. Substitute 20 g of saturated fat for x in the equation.

$y = 3.5 + 2.3x$	Original equation.
$y = 3.5 + 2.3(20)$	Substitute 20 for x.
$y = 49.5$	Multiply and add.

The model predicts that there would be 49.5 g of total fat in a burger with 20 g of saturated fat.

Health CONNECTION

Saturated and trans fats increase cholesterol and your risk of coronary heart disease. Trans fats, a result of hydrogenating or solidifying oil, have become increasingly common in processed foods—the U.S. Food and Drug Administration (FDA) required all foods to begin listing trans fat content beginning in January 2006. To learn more about trans fats, see www.keymath.com/DA .

d. Substitute 50 g of total fat for y in the equation.

$y = 3.5 + 2.3x$	Original equation.
$50 = 3.5 + 2.3x$	Substitute 50 for y.
$50 - 3.5 = 3.5 + 2.3x - 3.5$	Subtract 3.5 from both sides.
$46.5 = 2.3x$	Subtract.
$\frac{46.5}{2.3} = \frac{2.3x}{2.3}$	Divide both sides by 2.3.
$20.2 \approx x$	Reduce.

The model predicts that there would be about 20 g of saturated fat in a burger with 50 g of total fat.

Notice that you find the slope before the y-intercept when finding a line of fit. Because of the importance of slope, some mathematicians show it first. They use the **slope-intercept form** of a linear equation, often calling the slope m and the y-intercept b. This gives $y = mx + b$. Why is this equation equivalent to the intercept form that you have learned?

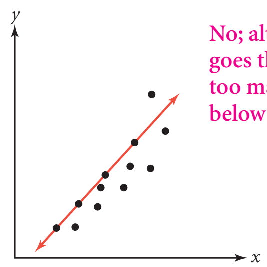
EXERCISES

You will need your graphing calculator for Exercise **4.**

Practice Your Skills

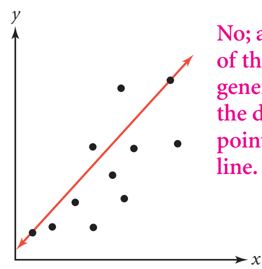
1. For each graph below, tell whether or not you think the line drawn is a good representation of the data. Explain your reasoning.

a.



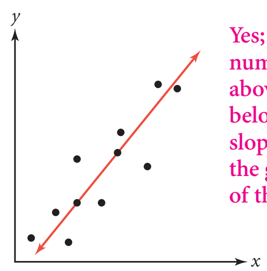
No; although this line goes through four points, too many points are below the line.

b. @



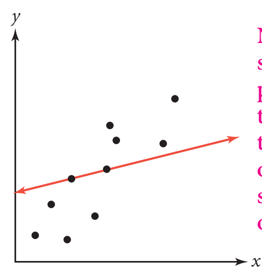
No; although the slope of the line shows the general direction of the data, too many points are below the line.

c.



Yes; about the same number of points are above the line as below the line, and the slope of the line shows the general direction of the data.

d. @



No; although the same number of points are above the line as below the line, the slope of the line doesn't show the direction of the data.

2. The line through the points (0, 5) and (4, 5) is horizontal. The equation of this line is $y = 5$ because the y-value of every point on it is 5. If a line goes through the points (2, -6) and (2, 8), what kind of line is it? What is its equation? vertical; $x = 2$

Slope-Intercept Form

[Ask] "Why can we now call the intercept b instead of a?" [The slope is whatever constant is multiplied by x, and the y-intercept is the constant being added to that product. You can represent those two constants with any pair of letters you choose, though some letters are more conventional to use than others.]

Closing the Lesson

You can use a line to model a set of data points for the sake of making predictions. The better the line fits the data, the better the predictions will be. If you have Fathom Dynamic Data Software, you can use student data to create a scatter plot and then use Fathom's movable line to picture possible **lines of fit.**

BUILDING UNDERSTANDING

Students acquire more practice with slopes and equations of lines, while gaining some experience with finding lines of fit.

ASSIGNING HOMEWORK

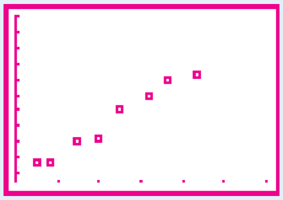
Essential	1–5, 10
Performance assessment	4
Portfolio	5, 7
Journal	1, 6, 8
Group	1, 5, 9
Review	11–13

Helping with the Exercises

Exercise 2 The explanation of equations for horizontal and vertical lines lays the groundwork for several exercises in this lesson and for exercises through the rest of the student text.

Exercise 4 **[Alert]** Be sure students convert years to months.

4a. There is a linear pattern.

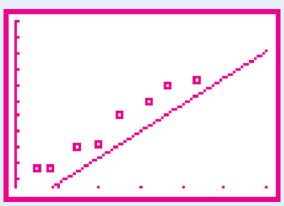


[0, 36, 6, 200, 1200, 100]

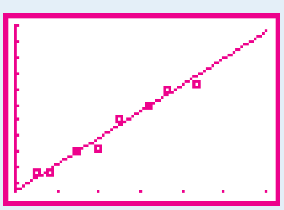
4b. Answers will vary. Using the points (8, 376) and (19, 684), the slope is 28.

4c. The slope represents the number of quarters Penny collects per month.

4d. $y = 28x$; the line needs to move up (the y-intercept needs to increase).



4e. A possible equation is $y = 152 + 28x$.

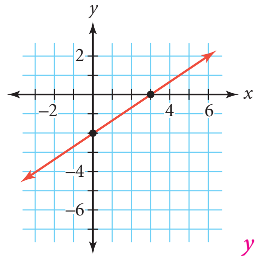


4f. The y-intercept represents the number of quarters Penny's grandmother gave her.

4g. Possible answer: 1160 quarters. The prediction may not be reliable because it extrapolates 10 months beyond the data.

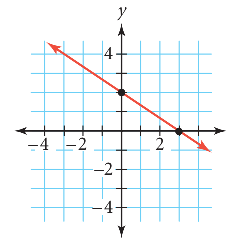
3. Write the equation of the line in each graph.

a. @



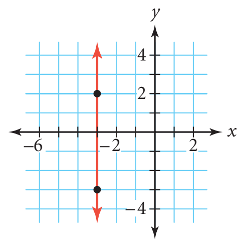
$y = -2 + \frac{2}{3}x$

b.



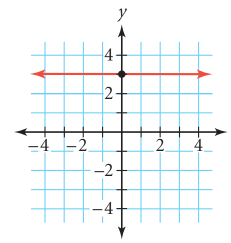
$y = 2 - \frac{2}{3}x$

c.



$x = -3$

d.



$y = 3$

4. On Penny's 15th birthday, her grandmother gave her a large jar of quarters. Penny decided to continue to save quarters in the jar. Every few months she counts her quarters and records the number in a table like this one. Predict how many quarters she'll have on her 18th birthday.

Penny's Savings

Number of months x	3	5	8	12	15	19	22	26
Number of quarters y	270	275	376	420	602	684	800	830

[Data sets: SAVMO, SAVQU]

a. Make a scatter plot of the data on your calculator. Is there a pattern?

b. Select two points through which a line of fit would pass. Find the slope of the line between these points.

c. What is the real-world meaning of the slope?

d. Use the slope you found in 4b to write an equation of the form $y = bx$. Graph this line on the scatter plot. What do you need to do to this line to better fit the data?

e. Estimate the y-intercept and write an equation in the form $y = a + bx$. Graph this new line. @

f. What is the real-world meaning of the y-intercept? @

g. Use your equation to predict how many quarters Penny will have on her 18th birthday.

Reason and Apply

5. **APPLICATION** A U.S. Census is conducted every ten years. One of the purposes of the Census is to measure each state's population in order to determine how many members each state will have in the House of Representatives for the next decade. Use the table to look for a relationship between a state's population and the number of members from that state in the House of Representatives.

Statistics for Some States

State	Estimated population, 2000 (millions)	Number of members in House of Representatives, 2001–2010	Number of members in Senate, 2001–2010
Alabama	4.4	7	2
Indiana	6.1	9	2
Michigan	9.9	15	2
Mississippi	2.8	4	2
North Carolina	8.0	13	2
Oklahoma	3.5	5	2
Oregon	3.4	5	2
Tennessee	5.7	9	2
Utah	2.2	3	2
West Virginia	1.8	3	2

(U.S. Bureau of the Census, in *Time Almanac 2004,* pp. 101–103, 177)
[Data sets: **STPOP, HREPS**]

The United States Constitution gives each state representation in the House of Representatives based on its population. To learn about historical methods of calculating representation, see www.keymath.com/DA . In the Senate, each state has equal representation regardless of size. This photo shows a joint session of both the House and the Senate.

a. Which statement makes more sense: The population depends on the number of members in the House of Representatives, or the number of members in the House of Representatives depends on the population? @

b. Based on your answer to 5a, define variables and make a scatter plot of the data. @

c. Find the equation of a line of fit. What is the real-world meaning of the slope? What is the real-world meaning of the y-intercept? @

d. The 2000 Census estimated California's population at 33.9 million. Use your equation to estimate the number of members California has in the House of Representatives.

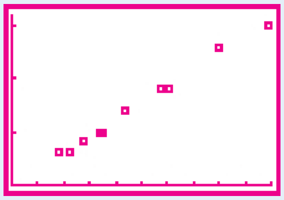
e. Minnesota has eight members in the House of Representatives. Use your equation to estimate the population of Minnesota.

f. You might find that a direct variation equation in the form $y = bx$ fits your data. Is this a reasonable model for the data? Explain why or why not. **The relationship should be a direct variation because it should go through the point (0, 0). A state with no population would have no representatives.**

Exercise 5 You might encourage students to research the laws dictating the number of members in the House of Representatives. The actual formula for determining representation is not linear, but a linear model fits fairly well. See www.keymath.com for links to sources. The first question gives an opportunity for more thinking about input and output variables.

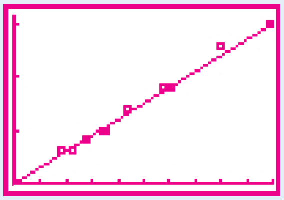
5a. The number of representatives depends on the population.

5b. Let x represent population in millions, and let y represent the number of representatives.



[0, 10, 1, 0, 16, 5]

5c. Answers will vary. Two possible points are (2.8, 4) and (6.1, 9). The slope between these points is approximately 1.5. The equation $y = 1.5x$ appears to fit the data with a y-intercept of 0. The slope represents the number of representatives per 1 million people. The y-intercept means that a state with no population would have no representatives.

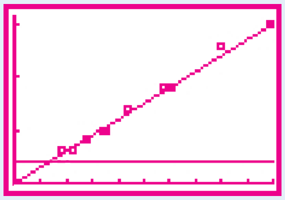


[0, 10, 1, 0, 16, 5]

5d. The equation $y = 1.5x$ gives $y = 1.5(33.9) = 50.85$, or 51 representatives. (For 2001–2010, California actually has 53 representatives.)

5e. The equation $y = 1.5x$ gives $8 = 1.5x$; $x = \frac{8}{1.5} = 5.\overline{3}$; 5.3 million. (The estimated population of Minnesota in the 2000 census was 4.9 million.)

6b. $y = 2$, where x represents population in millions and y represents the number of senators



Exercise 7 If the person walks at a truly constant rate, the data fit a line exactly. The slope and y-intercept of the line describe the data exactly.

8a. The slope is negative because the distance decreases as the time increases.

8b. The y-intercept represents the start distance for the walk; the x-intercept represents the time elapsed when the walker reaches the detector.

8c. Answers will vary. Quadrant II could indicate walking before you started timing. Quadrant IV could indicate that the walker walks past you; the distances behind you are considered negative.

Exercise 9 Be sure students realize that no single line satisfies all four conditions. Students can slide a slope triangle to work backward and find the y-intercept. This technique foreshadows the beginning of Lesson 4.3.

Exercise 10 You may need to say that a "family of lines" is a collection of lines that share some property. If students haven't worked on Exercise 2, they may need help with the equation of a vertical line in 10d. They may not recognize that c is a constant. Or they may not understand very deeply that the equation of a line gives a statement about both the x- and y-coordinates for points on that line. Pick a particular value for c and suggest that students name some points that satisfy that equation and draw the line connecting them. Then try it with one or two other values.

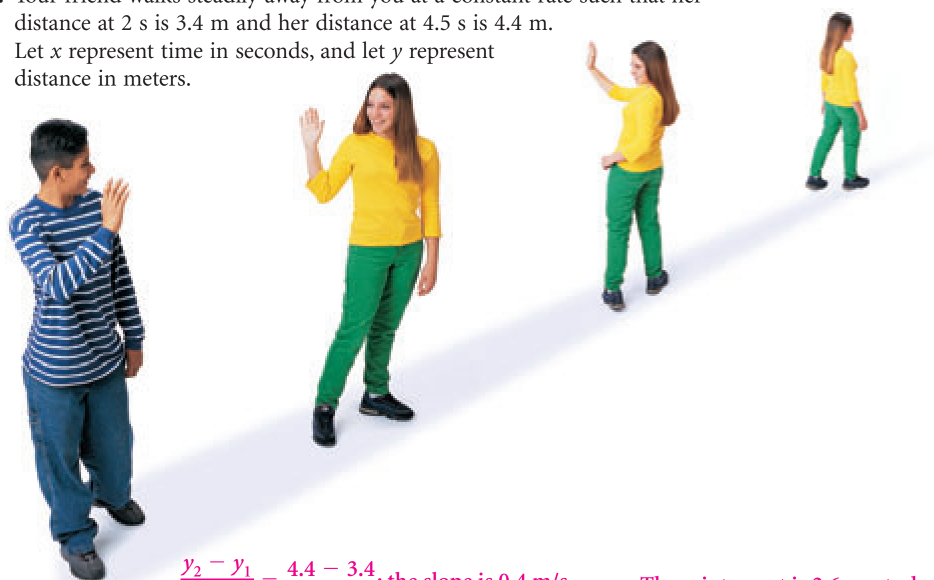
6. Use the table in Exercise 5 to answer these questions.

a. Does the population of a state affect its number of members in the Senate? No; each state has two senators regardless of its population.

b. Write an equation that models the number of senators from each state. Graph this equation on the same coordinate axes as 5c.

c. Describe the graph and explain why it looks this way. The graph is a horizontal line because there's no change in y, the number of senators.

7. Your friend walks steadily away from you at a constant rate such that her distance at 2 s is 3.4 m and her distance at 4.5 s is 4.4 m. Let x represent time in seconds, and let y represent distance in meters.



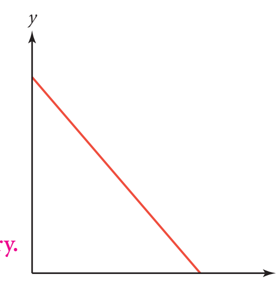
$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4.4 - 3.4}{4.5 - 2}$; the slope is 0.4 m/s.

a. What is the slope of the line that models this situation? ⓗ

b. What is the y-intercept of this line? Explain how you found it. The y-intercept is 2.6 m; students can find this by working backward with the slope or by estimating from a graph.

c. Write a linear equation in intercept form that models your friend's walk. $y = 2.6 + 0.4x$

8. Suppose this line represents a walking situation in which you're using a motion sensor to measure distance. The x-axis shows time and the y-axis shows distance from the sensor.



a. Is the slope positive, negative, zero, or undefined? Explain. ⓐ

b. What is the real-world meaning of the x- and y-intercepts? ⓐ

c. If the line extended into Quadrant II, what could that mean? If the line extended into Quadrant IV? ⓐ

9. Find the equation of a line that

a. Has a positive slope and a negative y-intercept. Answers will vary. $y = -8 + 4x$ is one possibility.

b. Has a negative slope and a y-intercept of 0. Answers will vary. $y = -2x$ is one possibility.

c. Passes through the points (1, 7) and (4, 10). $y = 6 + x$

d. Passes through the points (−2, 10) and (4, 10). $y = 10$

10. Each equation below represents a family of lines. Describe what the lines in each form have in common.

a. $y = a + 3x$ ⓐ **b.** $y = 5 + bx$ **c.** $y = a$ **d.** $x = c$

10a. All lines have a slope of 3; they are all parallel.

10b. All lines cross the y-axis at 5; they radiate around the point (0, 5).

10c. All lines are parallel to the x-axis, or horizontal.

10d. All lines are parallel to the y-axis, or vertical.

Review

2.5 **11.** For each of these tables of x- and y-values, decide if the values indicate a direct variation, an inverse variation, or neither. Explain how you made your decision. If the values represent a direct or inverse variation, write an equation.

a. @

x	y
−3	9
−1	1
−0.5	0.25
0.25	0.0625
7	49

b. @

x	y
−20	−5
−8	−12.5
2	50
10	10
25	4

c.

x	y
0	0
−6	15
8	−20
−12	30
4	−10

d.

x	y
78	6
31.2	2.4
−145.6	−11.2
14.3	1.1
−44.2	−3.4

3.6 **12.** Show the steps to solve each equation. Then use your calculator to verify your solution.

a. $8 - 12m = 17$ $\quad 8 - 12m = 17;\ -12m = 9;\ m = -0.75$

b. $2r + 7 = -24$ $\quad 2r + 7 = -24;\ 2r = -31;\ r = -15.5$

c. $-6 - 3w = 42$ $\quad -6 - 3w = 42;\ -3w = 48;\ w = -16$

1.2 **13.** Give the mean and median for each data set.

a. {1, 2, 4, 7, 18, 20, 21, 21, 26, 31, 37, 45, 45, 47, 48} mean: $24.8\overline{6}$; median: 21

b. {30, 32, 33, 35, 39, 41, 42, 47, 72, 74} mean: 44.5; median: 40

c. {107, 116, 120, 120, 138, 140, 145, 146, 147, 152, 155, 156, 179} mean: approximately 140.1; median: 145

d. {85, 91, 79, 86, 94, 90, 74, 87} mean: 85.75; median: 86.5

11a. neither

11b. inverse variation; $y = \frac{100}{x}$

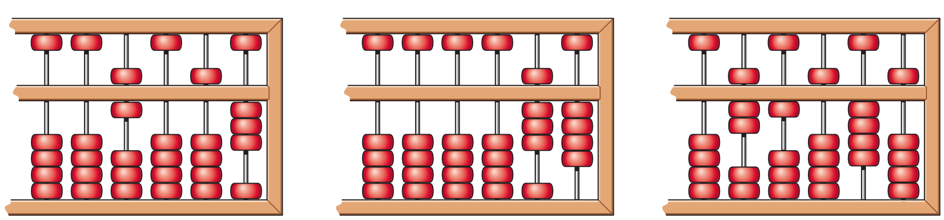
11c. direct variation; $y = -2.5x$

11d. direct variation; $y = \frac{1}{13}x$

Exercise 12 Students might use either the undoing or the balancing method.

IMPROVING YOUR VISUAL THINKING SKILLS

The traditional Japanese abacus, or *soroban,* is still widely used today. Each column shows a different place value—1, 10, 100, 1000, and so on. The four lower beads are moved up to represent the digits from 1 to 4. The fifth bead is moved down to show the digit 5. The digits 6 to 9 are shown with a combination of lower and upper beads. The first abacus below shows the number 6053.



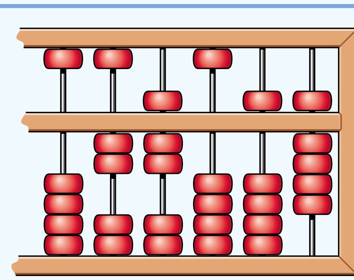
What numbers do the second and third abacuses show?

Sketch an abacus to show the number 27,059.

You can learn more about the abacus at www.keymath.com/DA .

IMPROVING VISUAL THINKING SKILLS

The second and third abacuses show 84 and 71,545, respectively; 27,059 would look like this:



LESSON

4.3

PLANNING

LESSON OUTLINE

One day:

10 min	Example
20 min	Investigation
5 min	Sharing
5 min	Closing
10 min	Exercises

MATERIALS

- Calculator Note 4A
- Sketchpad demonstration Point-Slope Form, *optional*

TEACHING

This lesson shows that the slope-intercept form of the equation of a line can be found from two points without having to find the y-intercept.

INTRODUCTION

The reference to homework is to Lesson 4.1, Exercise 7b. You can use the Sketchpad demonstration Point-Slope Form as an introduction to this lesson.

EXAMPLE

This example derives the point-slope form of the equation of a line. If the form happened to arise during Lesson 4.2, you may not need to spend much time on the example. Advise students who find it confusing that the investigation will make it clearer.

Encourage critical thinking by asking some questions. **[Ask]** “Do you think the situation is realistic? Do populations grow at a constant rate?” [Some do, but students may realize that population growth is often exponential up to a limit.]

LESSON

4.3 Point-Slope Form of a Linear Equation

Success breeds confidence.

BERYL MARKHAM

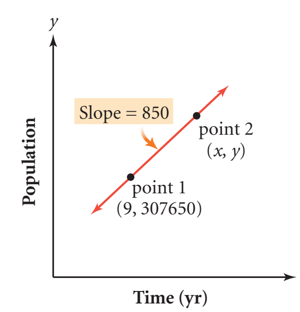
So far you have worked with linear equations in intercept form, $y = a + bx$. When you know a line’s slope and y-intercept, you can write its equation directly in intercept form. But what if you don’t know the y-intercept? One method that you might remember from your homework is to work backward with the slope until you find the y-intercept. But you can also use the slope formula to find the equation of a line when you know the slope of the line and the coordinates of only one point on the line.

EXAMPLE

Since the time Beth was born, the population of her town has increased at a rate of approximately 850 people per year. On Beth’s 9th birthday the total population was nearly 307,650. If this rate of growth continues, what will be the population on Beth’s 16th birthday?

▶ ***Solution***

Because the rate of change is approximately constant, a linear equation should model this population growth. Let x represent time in years since Beth’s birth, and let y represent the population.



In the problem, you are given one point, (9, 307650). Any other point on the line will be in the form (x, y). So let (x, y) represent a second point on the line. You also know that the slope is 850. Now use the slope formula to find a linear equation.

$$\frac{y_2 - y_1}{x_2 - x_1} = b$$ Slope formula.

$$\frac{y - 307{,}650}{x - 9} = 850$$ Substitute the coordinates of the point (9, 307650) for (x_1, y_1), and the slope 850 for b.

Because we know only one point, we use (x, y) to represent any other point.

LESSON OBJECTIVES

- Learn the point-slope form of an equation of a line
- Write equations in point-slope form that model real-world data

NCTM STANDARDS

CONTENT		PROCESS	
✓	Number		Problem Solving
✓	Algebra	✓	Reasoning
	Geometry		Communication
✓	Measurement		Connections
	Data/Probability	✓	Representation

Now solve the equation for y by undoing the subtraction and division.

$$y - 307{,}650 = 850(x - 9)$$ Multiply by $(x - 9)$ to undo the division.

$$y = 307{,}650 + 850(x - 9)$$ Add 307,650 to undo the subtraction.

The equation $y = 307{,}650 + 850(x - 9)$ is a linear equation that models the population growth. To find the population on Beth's 16th birthday, substitute 16 for x.

$$y = 307{,}650 + 850(x - 9)$$ Original equation.

$$y = 307{,}650 + 850(16 - 9)$$ Substitute 16 for x.

$$y = 313{,}600$$ Use order of operations.

The model equation predicts that the population on Beth's 16th birthday will be 313,600.

The equation $y = 307{,}650 + 850(x - 9)$ is a linear equation, but it is not in intercept form. This equation has its advantages too because you can clearly identify the slope and one point on the line. Do you see the slope of 850 and the point (9, 307650) within the equation? This form of a linear equation is appropriately called the **point-slope form.**

Point-Slope Form

If a line passes through the point (x_1, y_1) and has slope b, the **point-slope form** of the equation is

$$y = y_1 + b(x - x_1)$$

Investigation
The Point-Slope Form for Linear Equations

Silo and Jenny conducted an experiment in which Jenny walked at a constant rate. Unfortunately, Silo recorded only the data shown in this table.

Elapsed time (s) x	Distance to walker (m) y
3	4.6
6	2.8

Step 1 Find the slope of the line that represents this situation. -0.6 m/s

Step 2 Write a linear equation in point-slope form using the point (3, 4.6) and the slope you found in Step 1. $y = 4.6 - 0.6(x - 3)$

Step 3 Write another linear equation in point-slope form using the point (6, 2.8) and the slope you found in Step 1. $y = 2.8 - 0.6(x - 6)$

[Alert] Some students may be confused about choosing (x, y) to represent any point on the line. They may not yet grasp the idea that the equation relates coordinates of exactly those points lying on the line. Help them keep in mind the goal of coming up with such an equation.

[Alert] A few students may be confused about multiplying by $(x - 9)$. Remind them that they can consider $(x - 9)$ as a single number.

Resist simplifying $y = 307{,}650 + 850(x - 9)$ to $y = 300{,}000 + 850x$. Although the equations are equivalent, the latter is not in point-slope form. Students will learn the distributive property in Lesson 4.4.

You might ask students to go through the derivation again, using x_1, y_1, and b instead of the numbers.

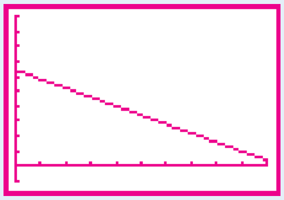
Emphasize that x_1, y_1, and b represent constants, whereas x and y represent variables. Also note that the coordinate x_1 is being subtracted from x.

Guiding the Investigation

One Step

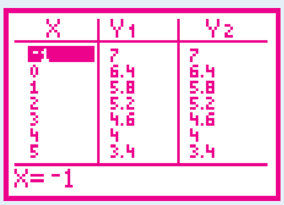
Direct students' attention to the Water Temperature table on page 236 and ask them to find a line of fit without finding the y-intercept.

Step 4 There appears to be only one line, which implies that the equations are equivalent.

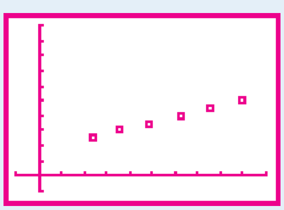


[0, 10, 1, −1, 10, 1]

Step 5 The Y_1- and Y_2-values are equivalent; again, this implies that the two seemingly different equations are equivalent.



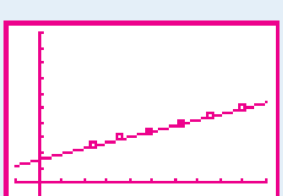
Step 6 Let x represent time in seconds, and let y represent temperature in degrees Celsius. The data set appears to have a linear pattern.



[−10, 100, 10, −10, 100, 10]

Steps 7 and 8 Each member of the group should be encouraged to select a different pair of points. Suggest that each group graph all of their lines on one calculator for easy comparison.

Step 8 Using the slope from Step 7, one possibility is $y = 35 + 0.38(x - 49)$.



Step 4 Enter the equation from Step 2 into Y_1 and the equation from Step 3 into Y_2 on your calculator, and graph both equations. What do you notice?

Step 5 Look at a table of Y_1- and Y_2-values. What do you notice? What do you think the results mean?

Now that you have some practice at writing point-slope equations, try using a point-slope equation to fit data.

The table shows how the temperature of a pot of water changed over time as it was heated.

Water Temperature

Time (s) x	Temperature (°C) y
24	25
36	30
49	35
62	40
76	45
89	50

Step 6 Define variables and plot the data on your calculator. Describe any patterns you notice.

Step 7 Answers will vary. Using (49, 35) and (62, 40), the slope is $\frac{5}{13}$.

Step 7 Choose a pair of points from the data. Find the slope of the line between your two points.

Step 8 Write an equation in point-slope form for a line that passes through your two points. Graph the line. Does your equation fit the data?

Step 9 Compare your graph to those of other members of your group. Does one graph show a line that is a better fit than the others? Explain. Answers will vary. Because the data are in such a tight linear pattern, there may not appear to be one better line of fit—they will all be pretty good.

If you look back at the investigation, you will notice that you found the point-slope form of a line even though you had only points (but not a slope) to start with. This is possible because you can still use the point-slope form when you know two points on the line; there's just one additional step. What is it? You must calculate the slope using the two points.

EXERCISES

You will need your graphing calculator for Exercises **3, 4, 5, 9,** and **10.**

Practice Your Skills

1. Name the slope and one point on the line that each point-slope equation represents.

a. $y = 3 + 4(x - 5)$ @ 4; (5, 3)

b. $y = 1.9 + 2(x + 3.1)$ 2; (−3.1, 1.9)

c. $y = -3.47(x - 7) - 2$ @ −3.47; (7, −2)

d. $y = 5 - 1.38(x - 2.5)$ −1.38; (2.5, 5)

2. Write an equation in point-slope form for a line, given its slope and one point that it passes through.

a. Slope 3; point (2, 5) $y = 5 + 3(x - 2)$

b. Slope −5; point (1, −4) $y = -4 - 5(x - 1)$

Step 9 **[Ask]** "How could you have wisely selected points in order to find the line of best fit to begin with?" [Choose points neither close together nor too far apart that appear to lie on a line that passes near most of the data.]

SHARING IDEAS

Choose students to present several different equations that have the same graphs. **[Ask]** "Is there a way to tell that the equations have the same graphs without actually graphing them?" Encourage all ideas. You don't need to answer this question. Students will get more experience identifying equivalent equations in Lesson 4.4.

Assessing Progress

You can assess students' understanding of input and output variables and their ability to find the slope of a line through two points and to graph data points and lines on a graphing calculator.

3. A line passes through the points $(-2, -1)$ and $(5, 13)$.

a. Find the slope of this line. ⓐ 2

b. Write an equation in point-slope form using the slope you found in 3a and the point $(-2, -1)$. ⓐ $y = -1 + 2(x + 2)$

c. Write an equation in point-slope form using the slope you found in 3a and the point $(5, 13)$. $y = 13 + 2(x - 5)$

d. Verify that the equations in 3b and c represent the same line. Enter the equations into Y1 and Y2 on your calculator, and compare their graphs and tables. The graphs coincide, and the tables are identical.

4. APPLICATION This table shows a linear relationship between actual temperature and approximate wind chill temperature when the wind speed is 20 mi/h.

Wind Chill with Wind Speed of 20 mi/h

Temperature (°F) x	5	10	15	20	25
Wind chill (°F) y	−15	−8.5	−2	4.5	11

a. Find the rate of change of the data (the slope of the line).

b. Choose one point and write an equation in point-slope form to model the data.

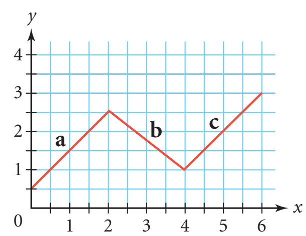
c. Choose another point and write another equation in point-slope form to model the data.

d. Verify that the two equations in 4b and c represent the same line. Enter the equations into Y1 and Y2 on your calculator, and compare their graphs and tables.

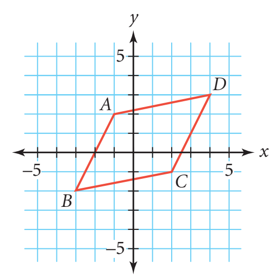
e. What is the wind chill temperature when the actual temperature is 0°F? What does this represent in the graph? −21.5°F; this is the graph's y-intercept.

5. Play the BOWLING program at least four times. [▶ See Calculator Note 4A for instructions on how to play the game. ◀] Each time you play, write down any equations you try and how many points you score.

Reason and Apply



6. The graph at right is made up of linear segments **a, b,** and **c.** Write an equation in point-slope form for the line that contains each segment. ⓗ



7. A **quadrilateral** is a polygon with four sides. Quadrilateral $ABCD$ is graphed at right.

a. Write an equation in point-slope form for the line containing each segment in this quadrilateral. Check your equations by graphing them on your calculator.

b. What is the same in the equations for the line through points A and D and the line through points B and C? What is different in these equations? ⓐ

c. What kind of figure does $ABCD$ appear to be? Do the results from 7b have anything to do with this? ⓐ *ABCD* appears to be a parallelogram because each pair of opposite sides is parallel; the equal slopes in 7b mean that $\overline{AD}$ and $\overline{BC}$ are parallel. $\overline{AB}$ and $\overline{DC}$ are parallel because they both have slope 2.

6a. $y = 0.5 + 1(x - 0)$ or $y = 2.5 + 1(x - 2)$

6b. $y = 2.5 - 0.75(x - 2)$ or $y = 1 - 0.75(x - 4)$

6c. $y = 1 + 1(x - 4)$ or $y = 3 + 1(x - 6)$

Exercise 7 [ELL] Students might not know the term *parallelogram.* Ask them to describe the figure if they cannot name it. Students will study the slopes of parallel and perpendicular lines in Lesson 11.1. If you need to cover these topics earlier, you may want to cover Lesson 11.1 immediately after Lesson 4.4.

7a. AD: $y = 2 + 0.2(x + 1)$ or $y = 3 + 0.2(x - 4)$
BC: $y = -2 + 0.2(x + 3)$ or $y = -1 + 0.2(x - 2)$
AB: $y = 2 + 2(x + 1)$ or $y = -2 + 2(x + 3)$
DC: $y = 3 + 2(x - 4)$ or $y = -1 + 2(x - 2)$

7b. The slopes are the same; the coordinates of the points are different.

Closing the Lesson

As needed, say that if you know two points on a line, you can find an equation for that line without finding the y-intercept. The form is called the **point-slope form.**

BUILDING UNDERSTANDING

Students work with the point-slope form of linear equations to model real-world data.

ASSIGNING HOMEWORK

Essential	1–3, 8, 9
Performance assessment	4, 8
Portfolio	10
Journal	6, 7
Group	5, 8, 10
Review	11–13

Helping with the Exercises

Exercise 1 [Alert] Some students may forget that the x-coordinate of the point is being subtracted from x. Therefore, in 1b, the x-coordinate of the point must be negative.

Exercises 3d and 4d These exercises foreshadow Lesson 4.4.

4a. $\frac{6.5}{5} = 1.3$; the data are exactly linear, so any two points will give this slope.

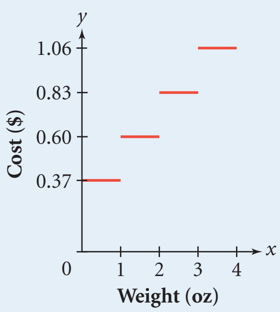
4b. Answers will vary. Using the point $(5, -15)$, the equation is $y = -15 + 1.3(x - 5)$.

4c. Answers will vary. Using the point $(20, 4.5)$, the equation is $y = 4.5 + 1.3(x - 20)$.

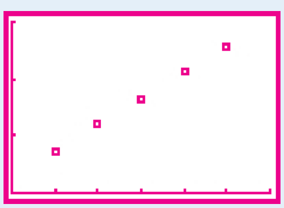
4d. The two equations should give the same graph and table.

Exercise 8 Letters or packages weighing more than 13 oz are subject to a different rate schedule. Therefore, the possible x-values for these data are restricted to whole numbers from 1 to 13. Research current postal rates through www.keymath.com/DA.

Bring up the idea of step functions. **[Ask]** "How could you graph this relationship accurately?"



8a. The data appear linear.



[0, 6, 1, 0, 1.5, 0.5]

8b. \$0.23/oz; this is the cost per additional ounce after the first.

8c. Answers will vary. Using the point $(1, 0.37)$, the equation is $y = 0.37 + 0.23(x - 1)$.

8e. The rates are given for weights not exceeding the given weights, so a letter weighing 3.5 oz would cost the same as a 4 oz letter, or \$1.06; a letter weighing 9.1 oz would cost the same as a 10 oz letter, or \$2.44.

8f. Answers will vary. A continuous line includes points whose x-values are not whole numbers and whose y-values are not possible rates.

9a. The data are approximately linear.

[0, 45, 5, 0, 650, 50]

8. APPLICATION The table shows postal rates for first-class U.S. mail in the year 2004.

a. Make a scatter plot of the data. Describe any patterns you notice.

b. Find the slope of the line between any two points in the data. What is the real-world meaning of this slope? @

c. Write a linear equation in point-slope form that models the data. Graph the equation to check that it fits your data points.

d. Use the equation you wrote in 8c to find the cost of mailing a 10 oz letter. \$2.44

e. What would be the cost of mailing a 3.5 oz letter? A 9.1 oz letter?

f. The equation you found in 8c is useful for modeling this situation. Is the graph of this equation, a continuous line, a correct model for the situation? Explain why or why not. @

Postal Rates

Weight not exceeding (oz) x	Cost (\$) y
1	0.37
2	0.60
3	0.83
4	1.06
5	1.29

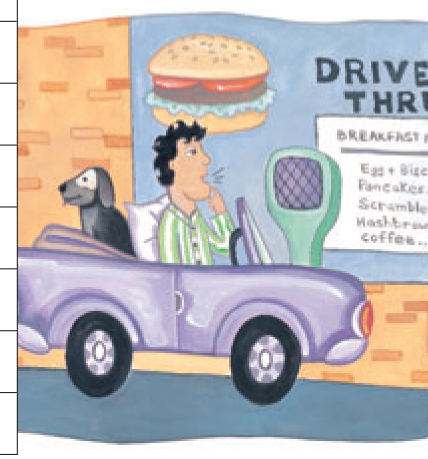
(U.S. Postal Service, *www.usps.com*)

9. APPLICATION The table below shows fat grams and calories for some breakfast sandwiches.

Nutrition Facts

Breakfast sandwich	Total fat (g) x	Calories y
Arby's Bacon 'n Egg Croissant	26	410
Burger King Croissanwich with Sausage, Egg & Cheese	39	520
Carl's Jr. Sunrise Sandwich	21	356
Hardee's Country Steak Biscuit	41	620
Jack in the Box Sourdough Breakfast Sandwich	26	445
McDonald's Sausage McMuffin with Egg	28	450
Sonic Sausage, Egg & Cheese Toaster	36	570
Subway Ham & Egg Breakfast Deli Sandwich	13	310

(*www.arbys.com, www.burgerking.com, www.carlsjr.com, www.hardeesrestaurants.com, www.jackinthebox.com, www.mcdonalds.com, www.sonicdrivein.com, www.subway.com*) [Data sets: **FFFAT, FFCAL**]



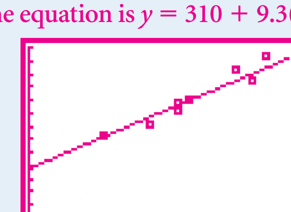
a. Make a scatter plot of the data. Describe any patterns you notice.

b. Select two points and find the equation of the line that passes through these two points in point-slope form. Graph the equation on the scatter plot.

c. According to your model, how many calories would you expect in a Hardee's Country Steak Biscuit with 41 grams of fat? $y = 310 + 9.3(41 - 13) = 570.4$; approximately 570 calories

d. Does the actual data point representing the Hardee's Country Steak Biscuit lie above, on, or below the line you graphed in 9b? Explain what the point's location means. The actual data point lies above the graph of $y = 310 + 9.3(x - 13)$; if a point lies above the line, the sandwich has more calories than the model predicts.

9b. Answers will vary. Using the points $(28, 450)$ and $(13, 310)$, the equation is $y = 310 + 9.3(x - 13)$.



[0, 45, 5, 0, 650, 50]

Exercise 9g Some students may misinterpret this to mean that all fat-free foods have 189 calories. Warn them that many factors influence calories.

e. Check each breakfast sandwich to find if its data point falls above, on, or below your line. Answers will vary. Using $y = 310 + 9.3(x - 13)$ as a model, three points are above the line, two points are on the line, and three points are below the line.

f. Based on your results for 9d and e, how well does your line fit the data?

g. If a sandwich has 0 grams of fat, how many calories does your equation predict? Does this answer make sense? Why or why not? Answers will vary. Using $y = 310 + 9.3(x - 13)$, approximately 189 calories; this makes sense, because not all calories in food come from fat.

10. APPLICATION This table shows the amount of trash produced in the United States in 1990 and 1995.

U.S. Trash Production

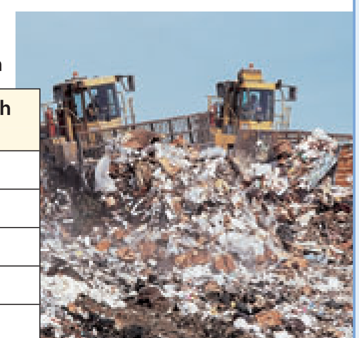
Year	Amount of trash (million tons)
1990	205
1995	214

(Environmental Protection Agency, *www.epa.gov*)

a. Let x represent the year, and let y represent the amount of trash in millions of tons for that year. Write an equation in point-slope form for the line passing through these two points. ⓐ

b. Plot the two data points and graph the equation you found in 10a. ⓐ

c. In 2000, 232 million tons of trash were produced in the United States. Plot this data point on the same graph you made in 10b. Do you think the linear equation you found in 10a is a good model for these data? Explain why or why not. ⓐ



This table shows more data about the amount of trash produced in the United States.

U.S. Trash Production

Year	Amount of trash (million tons)
1960	88
1965	103
1970	121
1975	128
1980	152
1985	164

(Environmental Protection Agency, *www.epa.gov*) [Data sets: TRYR, TRAMT]

d. Add these data points to your graph. Adjust the window as necessary.

e. Do you think the linear equation found in 10a is a good model for this larger data set? Explain why or why not.

f. Find the equation of a better-fitting line. ⓗ

g. Use your new equation from 10f to predict the amount of trash produced in 2010.

Answers will vary. Using $y = 200 + 3.7(x - 1990)$, 274 million tons.

Review

2.4 **11.** APPLICATION The volume of a gas is 3.50 L at 280 K. The volume of any gas is directly proportional to its temperature on the Kelvin scale (K).

a. Find the volume of this gas when the temperature is 330 K. 4.125 L

b. Find the temperature when the volume is 2.25 L. 180 K

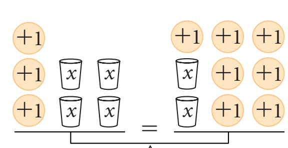
4.1 **12.** Find the slope of the line through the first two points given. Assume the third point is also on the line and find the missing coordinate.

a. $(-1, 5)$ and $(3, 1)$; $(5, \boxed{1})$ -1

b. $(2, -5)$ and $(2, -2)$; $(\boxed{2}, 3)$ undefined

c. $(-10, 22)$ and $(-2, 2)$; $(\boxed{0}, -3)$ $-\frac{5}{2}$

3.6 **13.** Write the equation represented by this balance. Then solve the equation for x using the balancing method. ⓐ



Exercise 11 Remind students of direct variations $y = kx$. You might say that Kelvin units are the same size as Celsius degrees but that 0 K is at about $-273°C$. It's called *absolute zero* because electrons at that temperature can't move. The word *degrees* is not used with the Kelvin scale.

Exercise 12 Encourage a variety of approaches. Students might graph, step over 1 unit at a time, draw slope triangles, or make calculator tables.

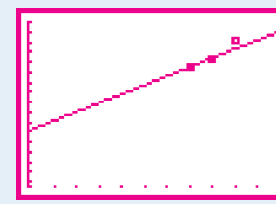
Exercise 13 Encourage students who are struggling to model the process using the balance. They may need to draw the steps. Don't rush students into solving equations with x's on both sides. They will see many of this type of problem in Chapter 5 in the context of solving systems of equations.

9f. Answers will vary. The line $y = 310 + 9.3(x - 13)$ appears to be a good fit.

Exercise 10 In 10e, a "good model" is one that allows accurate predictions. Part of the goal of 10g is to show that basing a model on a small set of data can lead to wild predictions. **[Ask]** "In what year does your equation predict that there were zero million tons of trash?" [Using $y = 214 + 1.8(x - 1995)$, the year would be 1876.] "Is this possible?"

10a. $y = 205 + 1.8(x - 1990)$ or $y = 214 + 1.8(x - 1995)$

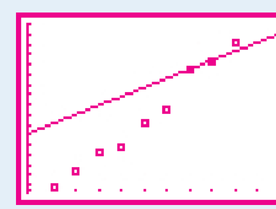
10b and 10c.



[1955, 2010, 5, 85, 250, 10]

The point (2000, 223) is somewhat close to the line, but the predicted value is too low.

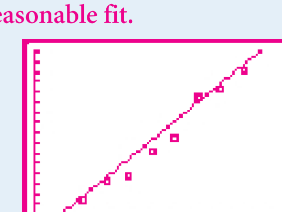
10d.



[1955, 2010, 5, 85, 250, 10]

10e. The data are generally linear, but the line doesn't fit them very well; a line with a steeper slope would be a better fit.

10f. Answers will vary. $y = 200 + 3.7(x - 1990)$ gives a reasonable fit.



[1955, 2010, 5, 85, 250, 10]

See page 723 for the answer to Exercise 13.

LESSON

4.4

PLANNING

LESSON OUTLINE

First day:

10 min Introduction

10 min Example A

25 min Investigation

5 min Sharing

Second day:

10 min Examples B and C

10 min Closing

30 min Exercises

MATERIALS

- Properties of Numbers (T), *optional*
- Properties of Equality (T), *optional*
- Sketchpad demonstration Distributive Property, *optional*

TEACHING

This lesson shows that linear equations are equivalent if their graphs or tables are the same or if they can be symbolically manipulated into the same equation.

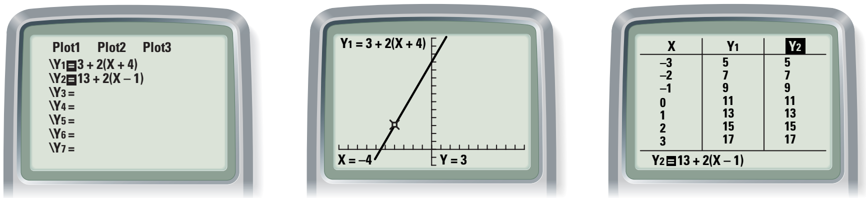
LESSON 4.4

Equivalent Algebraic Equations

In Lesson 4.3, you learned how to find an equation of a line through a given point. But a line goes through many points, so if you choose a different point, you'll get a different equation! In this lesson, you'll learn how to identify different equations that describe the same line.

These self-portraits of the American pop artist Andy Warhol (1928–1987) are like equivalent equations. Each screen-printed image is the same as the next, but Warhol's choice of colorization makes each look different.

For example, the line with slope 2 that passes through the point $(-4, 3)$ can be described by the equation $y = 3 + 2(x + 4)$. This line also passes through $(1, 13)$, so it can also be described by the equation $y = 13 + 2(x - 1)$. You can test that these equations are equivalent by graphing $Y_1 = 3 + 2(x + 4)$ and $Y_2 = 13 + 2(x - 1)$. The two equations graph the same line and give the same table values.



There are many different **equivalent equations** that can be used to describe any given line. In fact, both of the equations above can also be described in intercept form, $y = a + bx$. In this lesson you'll learn how to change equations to equivalent equations in intercept form by using mathematical properties and the rules for order of operations.

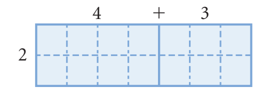
LESSON OBJECTIVES

- Learn and use the distributive property
- Rewrite equations to determine whether they are equivalent
- Formalize algebraic properties
- Identify properties as they are used in solving equations
- Introduce factoring as the reverse of the distributive property

NCTM STANDARDS

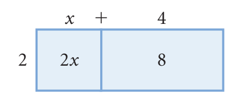
	CONTENT		PROCESS
✓	Number		Problem Solving
✓	Algebra	✓	Reasoning
✓	Geometry		Communication
	Measurement	✓	Connections
	Data/Probability	✓	Representation

The **distributive property** allows you to rewrite some expressions that contain parentheses. For an expression like $2(4 + 3)$, you can use the order of operations and add 4 and 3, then multiply this value by 2, to get 14. Or you can "distribute" the number outside the parentheses to all the numbers inside: $2(4 + 3) = 2 \cdot 4 + 2 \cdot 3$. This figure shows a model of the expression $2(4 + 3)$. You can think of the large rectangle either as a 2×7 rectangle or as a 2×4 rectangle and a 2×3 rectangle. The area is 14 no matter which way you compute it.



EXAMPLE A Use the distributive property to write $y = 3 + 2(x + 4)$ without parentheses.

▶ Solution Before adding 3, distribute the 2 through the sum of x and 4.



$y = 3 + 2(x + 4)$	Point-slope equation.
$y = 3 + 2 \cdot x + 2 \cdot 4$	Use the distributive property: Distribute 2 through $x + 4$.
$y = 3 + 2x + 8$	Multiply $2 \cdot 4$.
$y = 11 + 2x$	Combine like terms (add $3 + 8$).

So, $y = 3 + 2(x + 4)$ is equivalent to $y = 11 + 2x$. These are a point-slope equation and an intercept equation for the same line. What does each of the forms tell you about the line it describes?

The distributive property can be generalized like this:

Distributive Property

For any values of a, b, and c, this equation is true:

$$a(b + c) = a \cdot b + a \cdot c$$

In the investigation you'll further explore how to identify equivalent equations.

Investigation
Equivalent Equations

Here are six different-looking equations in point-slope form.

a. $y = 3 - 2(x - 1)$ **b.** $y = -5 - 2(x - 5)$ **c.** $y = 9 - 2(x + 2)$

d. $y = 0 - 2(x - 2.5)$ **e.** $y = 7 - 2(x + 1)$ **f.** $y = -9 - 2(x - 7)$

Step 1 Do the six equations represent the same line or different lines? Explain.

Step 1 Answers will vary. Some students might say that they cannot tell whether the equations are the same or different. Others may try graphing them and see that they are equivalent.

Step 2 Divide these equations among the members of your group. Use the distributive property to rewrite the right side of each equation. When you combine like terms, you should get an equation in intercept form. All equations become $y = 5 - 2x$.

Step 3 Enter your point-slope equation into Y_1, and enter your intercept equation into Y_2. Check that the two equations have the same calculator graph or table. How does this show that the equations are equivalent?

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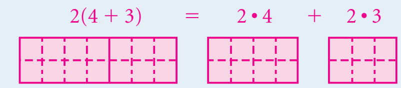
Step 2 Depending on the size of the group, each student may need to work with more than one equation.

Step 3 For all equations, the point-slope equation and the intercept equation should show identical graphs and tables, which means the same values satisfy both equations.

Distributive Property
Consider using algebra tiles to show several examples of the distributive property. The Sketchpad demonstration Distributive Property uses an algebra tile environment to explore the distributive property and factoring, which is introduced in Exercise 7.



EXAMPLE A
The equation $y = 3 + 2(x + 4)$ describes a line with slope 2 that passes through the point $(-4, 3)$, while the equation $y = 11 + 2x$ describes a line with slope 2 and y-intercept 11.

Guiding the Investigation

One Step
Point out the 15 equations listed on page 242 after Step 5 and ask students to categorize the lines by their equations. Encourage a variety of ways of checking for equivalence, focusing on graphing. Ask students how they could determine equivalence by looking at the expressions, and bring out the idea of the distributive property. Ask what other properties they know of that apply to operations on numbers, and list the properties of commutativity, associativity, and equality.

Step 1 **[Alert]** Watch for the claim that the lines are the same because their slopes are the same. Remind students that many parallel yet different lines can have the same slope. Students can compare the lines by graphing, by looking at tables, or by distributing and simplifying.

Step 6 Students might make four columns and place each equation in a column with equivalent equations. **[Alert]** Students may be confused by parts h and j. **[Ask]** "What's needed to get the equation in h or j into the same form as the others?" [For h, subtract $6x$ from both sides; for j, subtract $12x$ and divide by 2.] Students may forget to distribute the negative sign through part l and therefore may not be able to find an equivalent equation.

SHARING IDEAS

Ask for reports on Steps 5 and 6. If anyone disagrees about the report on Step 6, encourage discussion of the arguments rather than taking sides. If you announce a right answer, thinking about the problem might cease.

Introduce the term *standard form* to describe the equations in parts h and j. **[Ask]** "How would you generalize the standard form?" [The idea of generalizing may take some explaining. You might give the example of $y = a + bx$ as the generalized intercept form and elicit the idea that the standard form could be expressed as $ax + by = c$. Encourage representations that use other letters for the coefficients to communicate that the choice of letters is irrelevant.]

[Ask] "How many different equations in point-slope form might a line have?" [There are infinitely many, because a line contains infinitely many points.]

Ask what steps students took in transforming equations. Solicit a generalization of each and write down the property names from page 243, or use the two transparencies to point out the properties.

Step 4 Now, as a group, compare your intercept equations. What do the results show about the six equations? **All six point-slope equations can be transformed into $y = 5 - 2x$.**

Step 5 As a group, explain how you can tell that an equation in point-slope form is equivalent to one in intercept form. Think about how you can do this graphically and symbolically.

Step 5 **Answers will vary. You could graph the equations to see if they are the same line, or you could symbolically manipulate the point-slope form into intercept form.**

Here are fifteen equations. They represent only four different lines.

a. $y = 2(x - 2.5)$ **b.** $y = 18 + 2(x - 8)$

c. $y = 52 - 6(x + 8)$ **d.** $y = -6 + 2(x + 4)$

e. $y = 21 - 6(x + 4)$ **f.** $y = -14 - 6(x - 3)$

g. $y = -10 + 2(x + 6)$ **h.** $6x + y = 4$

i. $y = 11 + 2(x - 8)$ **j.** $12x + 2y = -6$

k. $y = 2(x - 4) + 10$ **l.** $y = 15 - 2(10 - x)$

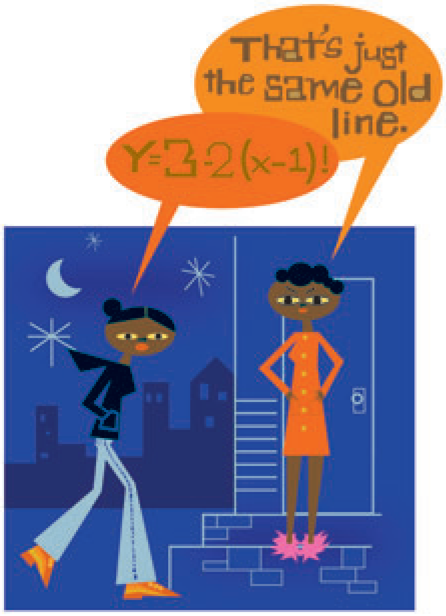
m. $y = 7 + 2(x - 6)$ **n.** $y = -6(x + 0.5)$

o. $y = -6(x + 2) + 16$

Step 6 **The intercept form for the equation of each line is given along with the letters of the equivalent equations.**
$y = -5 + 2x$: a, i, l, m
$y = 2 + 2x$: b, d, g, k
$y = -3 - 6x$: e, j, n
$y = 4 - 6x$: c, f, h, o

Step 6 Test your answer to Step 5 by finding the intercept form of each equation and then grouping equivalent equations.

Step 7 As a group, explain how you can tell that two equations in point-slope form are equivalent. **Students should recognize that intercept form is consistent, and therefore they should transform each point-slope form to intercept form.**



You have learned how to write linear equations in two different forms:

Intercept form	$y = a + bx$
Point-slope form	$y = y_1 + b(x - x_1)$

In the second part of the investigation, some of the equations had x and y on the same side, as in $12x + 2y = -6$. Equations in the form $ax + by = c$ are in **standard form.** What other equation in the investigation is in standard form?

No matter what form you start with, you can always rewrite any linear equation in intercept form. Then it's easy to recognize equivalent equations. Let's review properties that help you change the form of an equation.

For any values of a, b, and c, these properties are true:

Distributive Property

$a(b + c) = a(b) + a(c)$ Example: $6(-2 + 3) = 6(-2) + 6(3)$

Commutative Property of Addition

$a + b = b + a$ Example: $3 + 4 = 4 + 3$

Commutative Property of Multiplication

$ab = ba$ Example: $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{4} \cdot \frac{1}{2}$

Associative Property of Addition

$a + (b + c) = (a + b) + c$ Example: $2 + (1.5 + 3) = (2 + 1.5) + 3$

Associative Property of Multiplication

$a(bc) = (ab)c$ Example: $4\left(\frac{1}{3} \cdot 6.3\right) = \left(4 \cdot \frac{1}{3}\right)6.3$

There are also the properties that you have used to solve equations by balancing.

Properties of Equality

Given $a = b$, for any number c,

$a + c = b + c$	addition property of equality
$a - c = b - c$	subtraction property of equality
$ac = bc$	multiplication property of equality
$\frac{a}{c} = \frac{b}{c}\ (c \neq 0)$	division property of equality

EXAMPLE B Is the equation $y = 2 + 3(x - 1)$ equivalent to $6x - 2y = 2$?

▶ Solution

Use the properties to rewrite each equation in intercept form.

$y = 2 + 3(x - 1)$	Original equation.
$y = 2 + 3x - 3$	Distributive property (distribute 3 over $x - 1$).
$y = -1 + 3x$	Combine like terms.

So the intercept form of the first equation is $y = -1 + 3x$.

$6x - 2y = 2$	Original equation.
$-2y = 2 - 6x$	Subtraction property (subtract $6x$ from both sides).
$y = \frac{2 - 6x}{-2}$	Division property (divide both sides by -2).
$y = -1 + 3x$	Distributive property (divide each term by -2).

Assessing Progress

In their work on the investigation and in their presentations, students will demonstrate their ability to use the various properties of algebra, even if they don't yet know names for them.

The commutative and associative properties have been used informally up to this point. Students who have an intuitive sense of the properties may find it frustrating to have to justify each step. Encourage students to show their work clearly, but do not penalize students for doing easier steps in their head.

EXAMPLE B

This example repeats the ideas of the investigation and cites the relevant properties.

[Alert] Students may not understand how the distributive property applies to the fraction, which doesn't look like multiplication. You might review the term *multiplicative inverse* and use the multiplication property to write the equation as $y = -\frac{1}{2}(2 - 6x)$.

EXAMPLE C

Depending on students' comfort level with solving equations, you may want to show more detail than in this solution. For example, between the first two steps show $\frac{3x+4}{6} - 5 + 5 = 7 + 5$. **[Ask]** "Can the equation be solved a different way?" ["First multiply both sides by 6" is one possibility.]

Closing the Lesson

As needed, remind students that another form of a linear equation is the **standard form.** Linear equations are **equivalent** if their graphs are the same or if symbolic manipulation of one can give the other.

BUILDING UNDERSTANDING

Students practice using algebraic and equality properties to determine whether algebraic expressions are equivalent. They also encounter some simple factoring.

ASSIGNING HOMEWORK

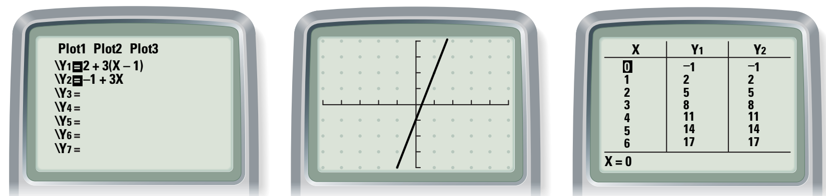
Essential	1–4, 7–10
Performance assessment	6, 12
Portfolio	12, 13
Journal	11
Group	3, 5
Review	14–16

Helping with the Exercises

Exercise 1 This exercise gives yet another way to check equivalence of linear equations, besides graphing, algebraic manipulation, and calculator tables. For linear equations, if two points match exactly, then the equations are equivalent.

The intercept form of the second equation is also $y = -1 + 3x$. So they are equivalent. You can also check that the intercept form and the point-slope form of the equation are equivalent by verifying that they produce the same line graph and have the same table of values. Unfortunately, you cannot enter the standard form into your calculator.

One of the authors, Jerald Murdock, works with two students.



$[-5, 5, 1, -4, 4, 1]$

EXAMPLE C Solve the equation $\frac{3x+4}{6} - 5 = 7$. Identify the property of equality used in each step.

▸ **Solution**

$\frac{3x+4}{6} - 5 = 7$	Original equation.
$\frac{3x+4}{6} = 12$	Addition property (add 5 to both sides).
$3x + 4 = 72$	Multiplication property (multiply both sides by 6).
$3x = 68$	Subtraction property (subtract 4 from both sides).
$x = 22\frac{2}{3}$	Division property (divide both sides by 3).

EXERCISES

You will need your graphing calculator for Exercises **1, 2,** and **10.**

Practice Your Skills

1. Is each pair of expressions equivalent? If they are not, change the second expression so that they are equivalent. Check your work on your calculator by comparing table values when you enter the equivalent expressions into Y1 and Y2.

a. $3 - 3(x + 4)$ $\quad 3x - 9$ @ $\quad$ not equivalent; $-3x - 9$

b. $5 + 2(x - 2)$ $\quad 2x + 1$ $\quad$ equivalent

c. $5x - 3$ $\quad 2 + 5(x - 1)$ $\quad$ equivalent

d. $-2x - 8$ $\quad -2(x - 4)$ $\quad$ not equivalent; $-2(x + 4)$ or $2(-x - 4)$

2. Rewrite each equation in intercept form. Show your steps. Check your answer by using a calculator graph or table.

a. $y = 14 + 3(x - 5)$ $y = -1 + 3x$ **b.** $y = -5 - 2(x + 5)$ @ $y = -15 - 2x$ **c.** $6x + 2y = 24$ $y = 12 - 3x$

3. Solve each equation by balancing and tell which property you used in each step.

a. $3x = 12$ $x = 4$; division property **b.** $-x - 45 = 47$ @ $-x = 92$; addition property $x = -92$; multiplication property

c. $x + 15 = 8$ $x = -7$; subtraction property **d.** $\frac{x}{4} = 28$ $x = 112$; multiplication property

4. Use the distributive property to rewrite each expression without parentheses.

a. $3(x - 2)$ $3x - 6$ **b.** $-4(x - 5)$ $-4x + 20$ **c.** $-2(x + 8)$ $-2x - 16$

5. An equation of a line is $y = 25 - 2(x + 5)$.

a. Name the point used to write the point-slope equation. ⓗ $(-5, 25)$

b. Find x when y is 15. $x = 0$

Reason and Apply

6. Solve each equation for the indicated variable.

a. $y = 3(x + 8)$ solve for x $x = \frac{y}{3} - 8$

b. $\frac{y - 3}{x - 4} = 10$ solve for y $y = 3 + 10(x - 4)$

c. $4(2y - 5) - 12 = x$ solve for y

7. In the expression $3x + 15$, the greatest common factor (GCF) of both $3x$ and 15 is 3. You can write the expression $3x + 15$ as $3(x + 5)$. This process, called **factoring,** is the reverse of distributing. Rewrite each expression by factoring out the GCF that will leave 1 as the coefficient of x. Use the distributive property to check your work.

a. $3x - 12$ @ $3(x - 4)$ **b.** $-5x + 20$ @ $-5(x - 4)$ **c.** $32 + 4x$ $4(8 + x)$ **d.** $-7x - 28$ $-7(x + 4)$

8. ***Mini-Investigation*** Consider the equation $y = 10 + 5x$ in intercept form.

a. Factor the right side of the equation. $y = 5(2 + x)$

b. Use the commutative property of addition to swap the terms inside the parentheses. $y = 5(x + 2)$

c. Your result should look similar to the point-slope form of the equation. What's missing? What is the value of this missing piece? @ The y_1-value is missing, which means it is zero; $y = 0 + 5(x + 2)$.

d. What point could you use to write the point-slope equation in 8c? What is special about this point? @ $(-2, 0)$; this is the x-intercept.

9. In each set of three equations, two equations are equivalent. Find them and explain how you know they are equivalent.

a. @
i. $y = 14 - 2(x - 5)$
ii. $y = 30 - 2(x + 3)$
iii. $y = -12 + 2(x - 5)$
Equations i and ii are equivalent.

b.
i. $y = -13 + 4(x + 2)$
ii. $y = 10 + 3(x - 5)$
iii. $y = -25 + 4(x + 5)$
Equations i and iii are equivalent.

c.
i. $y = 5 + 5(x - 8)$
ii. $y = 9 + 5(x + 8)$
iii. $y = 94 + 5(x - 9)$
Equations ii and iii are equivalent.

d.
i. $y = -16 + 6(x + 5)$
ii. $y = 8 + 6(x - 5)$
iii. $y = 44 + 6(x - 5)$
Equations i and iii are equivalent.

Exercise 2 Also encourage checks by substitution.

Exercise 3 Encourage variety. Students may solve 3a by dividing by 3 or multiplying by $\frac{1}{3}$. Similarly, 3b could involve division by -1 or multiplication by -1, and 3c could be solved by adding -15 or subtracting 15.

Exercise 6 If students are having trouble, suggest that they solve by undoing.

6c. $y = \dfrac{\frac{x + 12}{4} + 5}{2}$, or

$y = \frac{1}{8}x + 4$

Exercise 7 Make sure you assign this problem. **[Alert]** If students don't remember how to find the GCF, remind them that a number m is a *factor* of a number n if n is the product of m and some number. For example, 3 is a factor of 15 because 15 is the product of 3 and 5, and 3 is a factor of $3x$ because $3x$ is the product of 3 and x. So 3 is a common factor of 15 and $3x$. Factoring is mentioned again in the Chapter Review, Exercise 5.

10. The equation $3x + 2y = 6$ is in standard form.

a. Find x when y is zero. Write your answer in the form (x, y). What is the significance of this point? @ $x = 2$; the point $(2, 0)$ is the x-intercept.

b. Find y when x is zero. Write your answer in the form (x, y). What is the significance of this point? @ $y = 3$; the point $(0, 3)$ is the y-intercept.

c. On graph paper, plot the points you found in 10a and b and draw the line through these points. @

d. Find the slope of the line you drew in 10c and write a linear equation in intercept form. The slope is $-\frac{3}{2}$; $y = 3 - \frac{3}{2}x$.

e. On your calculator, graph the equation you wrote in 10d. Compare this graph to the one you drew on paper. Is the intercept equation equivalent to the standard-form equation? Explain why or why not.

f. Symbolically show that the equation $3x + 2y = 6$ is equivalent to your equation from 10d.

11. A line has the equation $y = 4 - 4.2x$.

a. Find the y-coordinate of the point on this line whose x-coordinate is 2. $y = -4.4$

b. Use the point you found in 11a to write an equation in point-slope form. $y = -4.4 - 4.2(x - 2)$

c. Find the x-coordinate of the point whose y-coordinate is 6.1. $x = -0.5$

d. Use the point you found in 11c to write a different point-slope equation. $y = 6.1 - 4.2(x + 0.5)$

e. Show that the point-slope equations you wrote in 11b and d are equivalent to the original equation in intercept form. Explain your procedure.

f. Is the point $(4, -12)$ on the line? How about $(-3, 16.6)$? Explain how you can determine whether a given point is on a line.

12. APPLICATION Dorine subscribes to an Internet service with a flat rate per month for up to 15 h of use. For each hour over this limit, there is an additional per-hour fee. The table shows data about Dorine's first two bills.

Internet Use

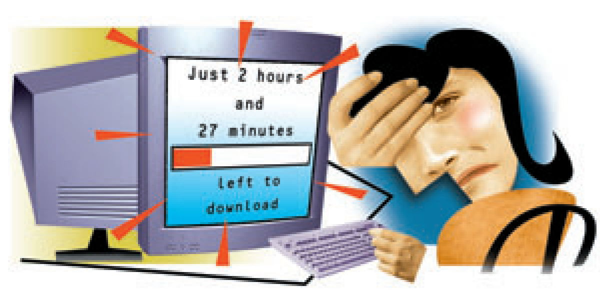
Month	Logged on (h)	Monthly fee ($)
January	20	15.20
February	23	17.75

a. Define your variables and use the data in the table to write an equation in point-slope form that models Dorine's total fee. @ $y = 15.20 + 0.85(x - 20)$

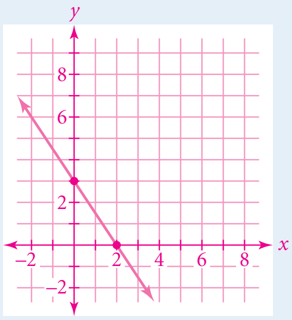
b. During March, Dorine was incorrectly charged $20 for being logged on for 25 h. What is her correct total fee? $19.45

c. In April, Dorine was logged on for 14 h. What was her total fee that month? Explain why you can't use your equation to answer this question. (*Hint:* Reread the problem carefully.)

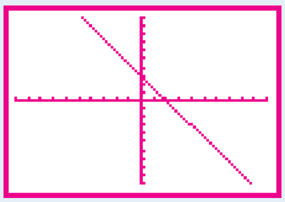
d. How many hours was Dorine logged on during a month when her fee was $23.70? 30 h



10c.



10e.



$[-10, 10, 1, -10, 10, 1]$

The two lines are the same; hence the equations are equivalent.

10f. See below.

11e. Answers will vary. You could rewrite each point-slope equation in slope-intercept form.

11f. $(4, -12)$ is not on the line; $(-3, 16.6)$ is on the line. Possible answer: Substitute the x- and y-values into the equation and check whether you get a true statement when you evaluate the equation. Or substitute the given x-value into the equation, evaluate, and see if it is equivalent to the given y-value.

12c. The equation is used to model the bill only when Dorine is logged on for more than 15 h. Substituting 15 for x gives the flat rate of $10.95 for all amounts of time less than or equal to 15 h.

10f.

$3x + 2y = 6$	Original equation.
$2y = 6 - 3x$	Subtract $3x$ from both sides.
$y = 3 - \frac{3}{2}x$	Divide both sides by 2.

13. On Saturday morning, Avery took a hike in the hills near her house. The table shows the cumulative number of calories she burned from the time she went to sleep Friday night until she finished her hike.

Avery's Hike

Time spent hiking (min)	Cumulative number of calories burned
5	568
10	591
15	614
20	637

a. Write a point-slope equation of a line that fits the data. ⓐ

b. Rewrite your equation from 13a in intercept form. $y = 545 + 4.6x$

c. What are the real-world meanings of the slope and the y-intercept in this situation? ⓗ

d. Could you use the point-slope equation $y = 821 + 4.6(x - 60)$ to model this situation? Explain why or why not.

e. What is the real-world meaning of the point used to write the equation in 13d?

Review

3.4 **14.** Moe Beel has a new cell phone service that is billed at a base fee of \$15 per month, plus 45¢ for each minute the phone is used. Consider the relationship between the time the phone is used and the total monthly cost. Let x represent time, in minutes, and let y represent cost, in dollars.

a. Give one point on the line, and state the slope of the line in dollars per minute. ⓐ possible answer: (0, 15); \$0.45/min

b. Write the equation of the line. Sketch its graph for the first 30 minutes.

c. How will the graph change if Moe adds Call Forwarding, changing the base fee to \$20? The line will be parallel to the original line, but 5 units higher.

d. How will the graph change if Moe drops Caller ID and Voice Mail so that there is no monthly base fee? The line will be parallel to the original line, but 15 units lower (passing through the origin).

e. How will the graph change if instead Moe adds the Text Messaging option, increasing his rate to 55¢ per minute? The line will be steeper but will have the same y-intercept.

4.1 **15.** Plot the points (4, 2), (1, 3.5), and (10, −1) on graph paper. These points are on the same line, or *collinear,* so you can draw a line through them.

a. Draw a slope triangle between (4, 2) and (1, 3.5), and calculate the slope from the change in y and the change in x.

b. Draw another slope triangle between (10, −1) and (4, 2), and calculate the slope from the change in y and the change in x.

c. Compare the slope triangles and the slopes you calculated. What do you notice?

d. What would happen if you made a slope triangle between (10, −1) and (1, 3.5)?

2.8 **16.** Show how to solve the equation $3.8 = 0.2(z + 6.2) - 5.4$ by using an undoing process to write an expression for z. Check your answer by substituting it into the original equation. $z = \frac{3.8 + 5.4}{0.2} - 6.2; z = 39.8$

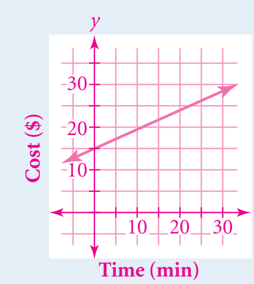
13a. The possible answers are $y = 568 + 4.6(x - 5)$; $y = 591 + 4.6(x - 10)$; $y = 614 + 4.6(x - 15)$; $y = 637 + 4.6(x - 20)$.

13c. The slope represents the number of calories burned per minute; the y-intercept represents the number of calories Avery burned from the time she went to sleep Friday night until she started hiking.

13d. Yes; it is equivalent to the slope-intercept equation $y = 545 + 4.6x$.

13e. The point (60, 821) tells you that if Avery hikes for 60 min, she will have burned a cumulative total of 821 calories since she went to sleep Friday night.

14b. $y = 15 + 0.45x$



Exercise 15 [Language] Emphasize that *collinear* means lying on the same line. Students will learn the definition of similar triangles in Chapter 11. Help them use terms such as *equal ratios* and *proportional* from Chapter 2 to describe the lengths of the sides of the slope triangles for 15c and d.

15a. $\frac{\text{change in } y}{\text{change in } x} = -\frac{1.5}{3} = -0.5$

15b. $\frac{\text{change in } y}{\text{change in } x} = -\frac{3}{6} = -0.5$

15c. Possible answers: The slope triangle side lengths for 15b are twice as long, but the slopes are equal.

15d. Possible answer: You would get a larger triangle, but the ratio of the side lengths would equal −0.5, giving a slope of −0.5.

LESSON

4.5

PLANNING

LESSON OUTLINE

One day:

25 min Investigation

5 min Sharing

5 min Closing

15 min Exercises

MATERIALS

- graph paper
- uncooked spaghetti, *optional*
- Fathom demonstration Life Expectancy, *optional*
- CBR demonstration Roll with It, *optional*

TEACHING

A linear equation in point-slope form, with the slope based on two points in a scatter plot, can help you make predictions.

Guiding the Investigation

The Fathom demonstration Life Expectancy can be used to replace this investigation.

One Step

Direct students' attention to the U.S. Life Expectancy at Birth table and ask students to predict the life expectancy of a person who will be born in 2022. Encourage a variety of approaches to finding lines of fit, but suggest choosing two data points that are not too close or too far apart. Urge students to express linear equations in point-slope form.

Step 1 Some students might call these "death predictions." Point out that life expectancy calculations don't predict when any particular person will die.

LESSON

4.5

Writing Point-Slope Equations to Fit Data

To give an accurate description of what has never occurred is the proper occupation of the historian.

OSCAR WILDE

In this lesson you'll practice modeling data that have a linear pattern with the point-slope form of a linear equation. You may find that using the point-slope form is more efficient than using the intercept form because you don't have to first write a direct variation equation and then adjust it for the intercept.

The development and improvement of vaccinations is one factor that has increased life expectancy over the decades.

Investigation

Life Expectancy

You will need

- graph paper

This table shows the relationship between the number of years a person might be expected to live and the year he or she was born. Life expectancy is a prediction that is very useful in professions like medicine and insurance.

U.S. Life Expectancy at Birth

Birth year	Female	Male	Combined
1940	65.2	60.8	62.9
1950	71.1	65.6	68.2
1960	73.1	66.6	69.7
1970	74.7	67.1	70.8
1975	76.6	68.8	72.6
1980	77.5	70.0	73.7
1985	78.2	71.2	74.7
1990	78.8	71.8	75.4
1995	78.9	72.5	75.8
2000	79.5	74.1	76.9

(National Center for Health Statistics, in *The World Almanac and Book of Facts 2004*, p. 76) [Data sets: **LEYR, LEFEM, LEMAL, LECOM**]

Step 1 Choose one column of life expectancy data—female, male, or combined. Let x represent birth year, and let y represent life expectancy in years. Graph the data points.

Step 2 Choose two points on your graph so that a line through them closely reflects the pattern of all the points on the graph. Use the two points to write the equation of this line in point-slope form.

Step 2 answers using the 1970 and 1990 data:
Female:
$y = 74.7 + 0.205(x - 1970)$
Male:
$y = 67.1 + 0.235(x - 1970)$
Combined:
$y = 70.8 + 0.23(x - 1970)$

Step 3 Graph the line with your data points. Does it fit the data?

Step 4 Use your equation to predict the life expectancy of a person who will be born in 2022.

Step 4 answer using the equations from Step 2: Female: 85.36 Male: 79.32 Combined: 82.76

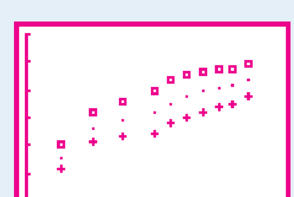
Step 5 Compare your prediction from Step 4 to the prediction that another group made analyzing the same data. Are your predictions the same? Are they close? Explain why it's possible to make different predictions from the same data.

Step 5 Predictions made from equations created with different points will vary, despite being from the same data set.

To help students make sense of the table, **[Ask]** "Why are the numbers increasing?" [Each year has brought better average living and working conditions and better food and medicine.]

Students can carry out this investigation on a calculator without graph paper. Be sure some groups work with the combined data.

Step 1



□ female
+ male
• combined

[1930, 2010, 10, 55, 85, 5]

Step 2 Suggest that students not choose points that are adjacent or at extremes of the data. Have some spaghetti ready for students who'd like to use it to indicate an easily adjustable line of fit.

Step 6 Slopes are around 0.2. This means that life expectancy increases by 0.2 yr each year, regardless of gender. Some students may notice that the slope of the data for males is slightly greater, which means that male life expectancy may eventually catch up with female life expectancy.

Step 6 Compare the slope of your line of fit to the slopes that other groups found working with different data sets. What does the slope for each data set tell you?

Step 7 As a class, select one line of fit that you think is the best model for each column of data—female, male, and combined. Graph all three lines on the same set of axes. Is it reasonable for the line representing the combined data to lie between the other two lines? Explain why or why not.

Step 8 How does the point-slope method of finding a line compare to the intercept-form method you learned about in Lesson 4.2? What are the strengths and weaknesses of each method?

Each student will have a different impression of the artwork in this museum. Similarly, different people can have different impressions of a set of data; this can result in different mathematical models.

You can summarize the point-slope method of fitting a line to the data like this: First, graph the data. Next, choose two points on a line that appears to show the direction of the data. Then, write the equation of the line.

Finally, you will need to graph the equation with the data and decide if the model is a good fit. With a wide scattering of points, there may be no pair of points from the existing data set that make a good model for the data. So you may need to adjust one or more of the three values in your equation $(x_1, y_1,$ or $b)$ to improve the the model. Exercise 8 will give you a chance to experiment with these changes.

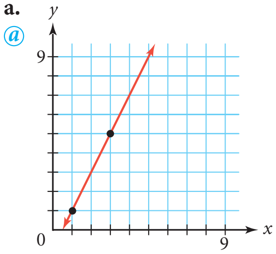
EXERCISES

You will need your graphing calculator for Exercises **3, 4, 5, 6, 8,** and **9.**

Practice Your Skills

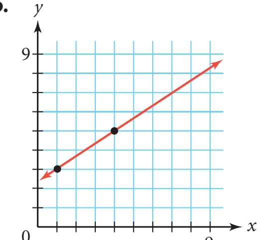
1. Write the point-slope form of the equation for each line graphed below.

a. @



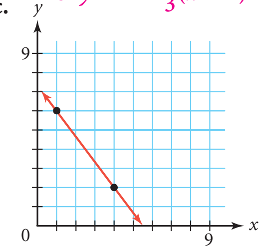
$y = 1 + 2(x - 1)$ or $y = 5 + 2(x - 3)$

b.



$y = 3 + \frac{2}{3}(x - 1)$ or $y = 5 + \frac{2}{3}(x - 4)$

c.

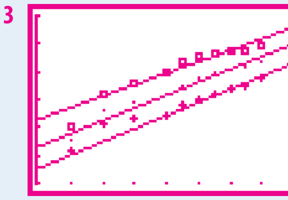


$y = 6 - \frac{4}{3}(x - 1)$ or $y = 2 - \frac{4}{3}(x - 4)$

2. Look at each graph in Exercise 1 and estimate the y-intercept. Then convert your point-slope equations to intercept form. How well did you estimate? ⓗ

$y = -1 + 2x$ $y = \frac{7}{3} + \frac{2}{3}x$ or $y = 2.\bar{3} + 0.\bar{6}x$ $y = \frac{22}{3} - \frac{4}{3}x$ or $y = 7.\bar{3} - 1.\bar{3}x$

Step 3



[1930, 2010, 10, 55, 85, 5]

The graph fits the data reasonably well.

Step 7 Use a calculator projection panel if you have one. Point out that the chart shows the combined life expectancy. **[Ask]** "Is the combined data simply an average of the male and female numbers?" [no] "Why not?" [Averaging averages doesn't work when the sizes of data sets are different. The number of men and the number of women are not exactly the same.]

SHARING IDEAS

Ask students how accurate they think their prediction is. Have them discuss the number of years they think their linear model will give a reliable prediction.

Point out the quotation from Oscar Wilde at the beginning of the lesson. Ask the class to brainstorm methods that mathematicians use to "give an accurate description of what has never occurred." Besides using lines of fit to make predictions, students might raise ideas such as that the "perfect" geometric figures of mathematics never occur and that perfectly accurate measurements (except when counting discrete objects) are impossible.

You can use the CBR demonstration Roll with It to give students more practice modeling data with point-slope equations.

Assessing Progress

Look for students' abilities at making scatter plots, finding the slope of a line through two points, and evaluating an equation at a point. Also check their understanding of the intercept form and their skill at comparing and contrasting different approaches.

See page 723 for answers to Steps 7 and 8.

NCTM STANDARDS

CONTENT		PROCESS	
✓	Number	✓	Problem Solving
✓	Algebra	✓	Reasoning
	Geometry	✓	Communication
	Measurement	✓	Connections
✓	Data/Probability	✓	Representation

LESSON OBJECTIVES

- Write linear equations in point-slope form that model real-world data
- Discover strengths of the point-slope form for linear equations
- Learn to deal with variation in linear data

Closing the Lesson

Say that you can use two points in a data set to find the point-slope form of a linear equation and then use it to make predictions.

BUILDING UNDERSTANDING

The exercises involve using the point-slope form of linear equations to fit data and make predictions.

ASSIGNING HOMEWORK

Essential	1–5, 8
Performance assessment	7, 8
Portfolio	8
Journal	3, 5
Group	4–6
Review	9–11

Helping with the Exercises

Exercise 3 Remind students that the *x-intercept* is the point where the line crosses the *x*-axis. This exercise previews the factored form of polynomials that will be introduced in Lesson 9.4.

Exercise 4 Students might choose to use as input values the number of years since 1900 or 1950 or 1976. Discuss 4d with students. They should be aware that mathematical models like this one, based on a short time period of about 30 yr, cannot necessarily be used to make conclusions about the distant past or future.

4a. Answers will vary. Using the points (1982, 341) and (1996, 363) gives the equation $y = 341 + 1.6(x - 1982)$, where x is the year and y is the concentration of CO_2 in parts per million.

3. Graph each linear equation on your calculator and name the x-intercept. Make a conjecture about the x-intercept of any equation in the form $y = b(x - x_1)$.

a. $y = 2(x - 3)$ @ 3 **b.** $y = \frac{1}{3}(x + 4)$ −4 **c.** $y = -1.5(x - 6)$ 6

The x-intercept of $y = b(x - x_1)$ is at $x = x_1$.

4. APPLICATION Carbon dioxide is one of several greenhouse gases that is emitted into the atmosphere from a variety of sources, including automobiles. The table shows the concentration of carbon dioxide (CO_2) in the atmosphere measured from the top of Mauna Loa volcano in Hawaii each January. The concentration of CO_2 is measured in parts per million (ppm).

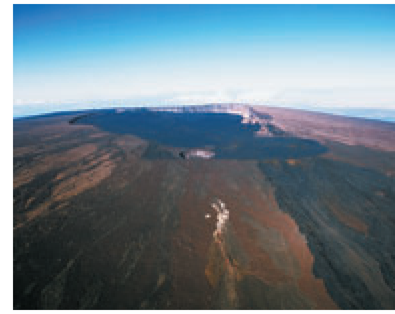
a. Define variables and write an equation in point-slope form that models the data.

b. Graph your equation to confirm that the line fits the data.

c. Use your equation to predict what the concentration of CO_2 will be in 2020.

d. What would be the x-intercept for your equation? Does its real-world meaning make sense? Explain why or why not.

e. According to your equation, what is the typical change in CO_2 concentration per year? about 1.6 ppm/yr



Mauna Loa is the largest and most active volcano on Earth. Research on Mauna Loa has revealed a great deal about global changes in the atmosphere. For more information about the causes and effects of the increase in atmospheric CO_2, see www.keymath.com/DA .

CO_2 Concentration

Year	CO_2 (ppm)
1976	332
1978	336
1980	339
1982	341
1984	344
1986	347
1988	351
1990	354
1992	356
1994	359
1996	363
1998	367
2000	369
2002	373

(Carbon Dioxide Information Analysis Center, *cdiac.esd.ornl.gov*) [Data sets: **CO2YR, CO2CN**]

Reason and Apply

5. APPLICATION Alex collected this table of data by using two thermometers simultaneously. Alex suspects that one or both of the thermometers are somewhat faulty.

a. Graph the data. @

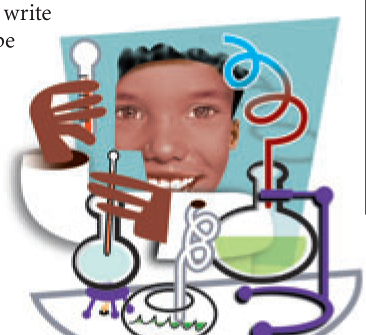
b. Write an equation in point-slope form that models Alex's data. @

c. Graph your equation to confirm that the line fits the data.

d. The freezing point of water is 0°C, which is equivalent to 32°F. The boiling point of water is 100°C, which is equivalent to 212°F. Use this information to write another equation in point-slope form that models the true relationship between the Celsius and Fahrenheit temperature scales. @

e. Write the equations from 5b and d in intercept form. Are they equivalent? @

f. Do you think that Alex's thermometers are faulty? Explain why or why not. The difference could be a result of measurement error or faulty procedures.

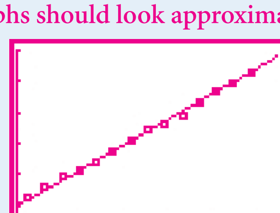


Temperature Readings

Celsius (°C) x	Fahrenheit (°F) y
14.5	55.0
20.0	67.0
28.4	86.7
39.5	105.6
32.3	87.1
29.0	81.6
26.2	82.3
25.7	75.2
31.2	88.6

[Data sets: **TEMPC, TEMPF**]

4b. All graphs should look approximately like this



[1975, 2005, 5, 325, 380, 10]

4c. The equation $y = 341 + 1.6(x - 1982)$ gives 402 ppm.

4d. Using the equation in 4c, the x-intercept is about 1769. It represents the year when the concentration of CO_2 would have been zero. This is not reasonable, because plants depend on CO_2 and there would have been some concentration of CO_2 for as long as there have been plants. The model is limited; it cannot be extended much before or after the time period of the data.

Exercise 5 If you did the CBL 2 demonstration Heating Up with your class, this exercise would make an interesting follow-up.

6. **APPLICATION** The table lists the concentration of dissolved oxygen (DO) in parts per million at various temperatures in degrees Celsius from a sample of lake water.

a. Graph the data.

b. Write an equation in point-slope form that models the data.

c. Graph your equation to confirm that the line fits the data.

d. Use your equation to predict the concentration of dissolved oxygen in parts per million when the water temperature is 2°C.

e. Use your equation to predict the water temperature in degrees Celsius when the concentration of dissolved oxygen is 12 ppm.
Using $y = 11 - 0.6(x - 13)$, the temperature is about 11.3°C.

Dissolved Oxygen

Temperature (°C) x	DO (ppm) y
17	8
15	9
13	11
16	10
11	14
13	11
10	14
8	14
6	16
7	13
8	14
4	17
5	15
9	13
6	16

[Data sets: **DOTMP, DOPPM**]

7. Use the data and the equation you found in Exercise 6.

a. Write an equation with the same slope that passes through the point farthest above the line. Using the slope -0.6, $y = 14 - 0.6(x - 11)$.

b. Write an equation with the same slope that passes through the point farthest below the line. $y = 13 - 0.6(x - 7)$

c. Rewrite all three equations in intercept form.

d. Based on your answer to 7c, how accurate are predictions made using your equation from Exercise 6 likely to be? ⓗ
The equation has prediction accuracy within 1.8 ppm.

8. ***Mini-Investigation*** Scoop has a rolling ice cream cart. He recorded his daily sales for the last seven days and the mean daytime temperature for each day.

Ice Cream Sales

Day	1	2	3	4	5	6	7
Temperature (°F)	83	79	75	70	71	67	62
Sales (cones)	66	47	51	23	33	30	21

[Data sets: **ICTMP, ICSAL**]

a. Find the equation of the line that passes through the points (79, 47) and (67, 30). (Use the second point as the point in the point-slope form.) ⓐ $y = 30 + 1.4(x - 67)$

b. Graph the data and your line from 8a on your calculator. Sketch the result.

You should have noticed in 8b that the line does not fit the data well. In fact, no two points from this data set make a good model. In 8c–e you'll adjust the values of y_1 and b in $y = y_1 + b(x - x_1)$ to find a better model.

c. Copy the table shown, and begin by changing the value of y_1. Write two new equations, one with a larger value for y_1 and one with a smaller value for y_1. Graph each equation, and describe how the graphs compare to your original equation. ⓐ

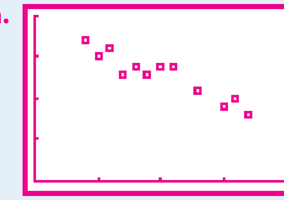
Value	Increase	Decrease
y_1		
b		

d. Now write two new equations that have the same values of x_1 and y_1 as the original, but larger and smaller values of b. Graph each equation, and describe how the graphs compare to your original equation.

e. Continue to adjust your values for y_1 and b until you find a line that fits the data well. Record your final equation. Graph your equation with the data and sketch the result.

Exercise 6 **[Ask]** "Why does the graph of the data show only 12 points?" [Three are double points: (6, 16), (8, 14), (13, 11).]

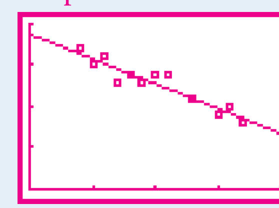
6a.



[0, 20, 5, 0, 20, 5]

6b. Using (13, 11) and (8, 14), the equation is $y = 11 - 0.6(x - 13)$ or $y = 14 - 0.6(x - 8)$.

6c. one possible answer:



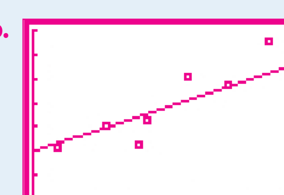
6d. Using $y = 11 - 0.6(x - 13)$, the concentration of dissolved oxygen is 17.6 ppm.

7c. $y = 18.8 - 0.6x$;
$y = 20.6 - 0.6x$;
$y = 17.2 - 0.6x$

Exercise 7d **[Alert]** Students may claim that the line can predict within 1.6 ppm because the lower line is within 1.6 ppm of the modeling line. Bring out the fact that the line is not within 1.6 ppm of all the data points, but it is within 1.8 ppm.

Exercise 8 This would be a good exercise for movable lines in Fathom.

8b.

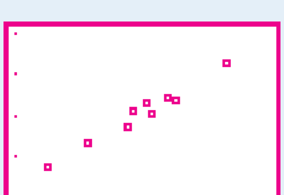


[60, 85, 5, 0, 70, 10]

8c. Equations will vary. The graph with a larger y_1-value is parallel but higher, and the graph with a smaller y_1-value is parallel but lower.

See page 252 for answers to Exercises 8d and 8e.

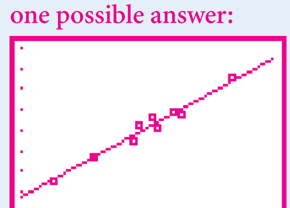
5a.



[10, 45, 5, 40, 120, 20]

5b. Using the points (20, 67) and (31.2, 88.6), the slope is approximately 1.9 and a possible equation is $y = 67 + 1.9(x - 20)$.

5c. one possible answer:

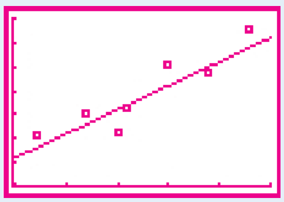


5d. $y = 32 + 1.8(x - 0)$ or $y = 212 + 1.8(x - 100)$

5e. The sample equation in 5b gives $y = 29 + 1.9x$; the equations in 5d both give $y = 32 + 1.8x$.

8d. Equations will vary. The graphs pass through the point (67, 30), but the one with a larger value of b is steeper and the one with a smaller value of b is less steep.

8e. possible equation: $y = 26 + 2(x - 67)$



[60, 85, 5, 0, 70, 10]

9c. After 45 full days, there will be only one biscuit left, so the box will be empty at some time on the 46th day.

9d. When the box was new, before Anchor had any biscuits, there were 136 biscuits.

Review

4.3 **9.** **APPLICATION** Bryan has bought a box of biscuits for his dog, Anchor. Anchor always gets three biscuits a day. At the start of the 10th day after opening the box, Bryan counts 106 biscuits left. Let x represent the number of days after opening the box, and let y represent the number of biscuits left.

a. In a graph of this situation, what is the slope? ⓗ -3

b. Write a point-slope equation that models the situation. $y = 106 - 3(x - 10)$

c. When will the box be empty?

d. What is the real-world meaning of the y-intercept?

2.8 **10.** Solve the equation $2x - 3(y + 1) = 12$ for y by copying and filling in this table. @

Description	Undo	Equation
Pick y.		$y =$
$+1$	-1	
$\cdot(-3)$	$/(-3)$	
$+2x$	$-2x$	

$y = \frac{12 - 2x}{-3} - 1$, or $y = -5 + \frac{2x}{3}$

$y + 1 = \frac{12 - 2x}{-3}$, or $y + 1 = -4 + \frac{2x}{3}$

$-3(y + 1) = 12 - 2x$

$2x - 3(y + 1) = 12$

11. You've worked with various types of problems involving rates. A new kind of problem that uses rates is called a **work problem.** In a work problem, you usually know how long it would take someone or something to complete an entire job. You use the reciprocal of the complete time to find a rate of work. For example, if Mavis paints 1 entire room in 10 hours, she paints $\frac{1}{10}$ of the room each hour. These problems rely on the formula *rate of work* · *time* = *part of work*. These problems also assume that a complete job is equivalent to 1.

Mavis and Claire work for a house painter. Mavis can paint a room in 10 hours, and Claire can paint a room in 8 hours. How long will it take them to paint a room if they work together?

Let t represent the number of hours that Mavis and Claire paint. Mavis paints $\frac{1}{10}$ of a room each hour, and Claire paints $\frac{1}{8}$ of a room each hour. So you can write the equation $\frac{1}{10}t + \frac{1}{8}t = 1$.

a. Solve this equation, check your answer, and state the solution. $4.\overline{4}$ h, or about 4 h 27 min

b. Solve this problem using a similar procedure: When fully turned on, the faucet of a bathtub fills a tub in 30 minutes. When the tub is full of water and the drain is opened, the tub empties in 45 minutes. If the faucet is fully turned on *and* the drain is open at the same time, how much time does it take to fill the tub? 90 min

LESSON

4.6

More on Modeling

When you can measure what you are talking about and express it in numbers, you know something about it.

LORD KELVIN

Several times in this chapter you have found the equation of a representative line to fit data. Making, analyzing, and using predictions based on equation models is important in the real world. For this reason it is often helpful and even important that different people arrive at the same model for a given set of data. For this to happen, each person must get the same slope and y-intercept. To do that, they have to follow the same systematic method.

Statisticians have developed many methods of finding a line or curve that fits a set of data well. In this lesson you'll learn a method that uses the quartiles you learned about in Chapter 1.

Investigation
Bucket Brigade

You will need

- a stopwatch
- a bucket
- graph paper

In this investigation you will use a systematic method for finding a particular line of fit for data.

Procedure Note

Select a class member as timer. Everyone should line up single file. Your line might wrap around the room. Spread out so that there is an arm's length between two people.

Step 1 Line up in a bucket brigade. (See the Procedure Note.) Record the number of people in the line. Starting at one end of the line, pass the bucket as quickly as you can to the other end. Record the total passing time from picking up the bucket to setting it down at the very end.

Step 2 Now have one or two people sit down and close up the gaps in the line. Repeat the bucket passing. Record the new number of people and the new passing time.

Step 3 Continue the bucket brigade until you have collected 10 data points in the form (*number of people, passing time in seconds*).

LESSON

4.6

PLANNING

LESSON OUTLINE

First day:

50 min	Investigation (Steps 1–12)

Second day:

10 min	Investigation (Step 13)
10 min	Sharing
10 min	Example
5 min	Closing
15 min	Exercises

MATERIALS

- stopwatch or watch with second hand
- bucket or other object to pass
- graph paper
- Calculator Notes 1D, 4B
- Fathom demonstration More on Modeling, *optional*

TEACHING

This lesson builds on the five-number summaries introduced in Lesson 1.3. If you didn't cover Lesson 1.3, you can still cover Lessons 4.6–4.8, but you'll need to introduce five-number summaries first. One way to standardize the choice of two points through which a line of fit passes is to use the first and third quartiles of each data variable.

Guiding the Investigation

One Step

Using some data (perhaps from a bucket brigade such as in the first part of the investigation or from the example), point out that so far students have come up with different valid modeling lines. Challenge groups to develop systematic methods, based on statistics of the two data sets, for

producing these lines. As you circulate, remind students that the mean is overly sensitive to outliers. Some groups might develop something like the median-median line, but encourage groups that are stuck to produce two points through which the line will pass. The "middle of each half" is a natural idea that leads to Q-points.

Step 1 **[ELL]** A *bucket brigade* is a line of people passing a bucket, usually full of water to help put out a fire.

In place of a bucket, students can use any object they can pass. They might run each brigade several times and average the times. In this unusual situation, time is the output rather than the input variable. **[Ask]** "Which variable depends on which other variable?" [The time depends on how far the bucket needs to be passed.]

Step 2 Students who have just sat down might do the timing, record the data, or begin to make scatter plots. You can also vary the size of groups that sit down.

Step 5 Refer students to Lesson 1.3 or Calculator Note 1D to review five-number summaries.

Step 7 If students have difficulty making a vertical box plot, encourage them to make a horizontal one and then rotate it.

Step 11 The slope represents how long it takes one person to pass the bucket. The y-intercept is the time it takes for no people to pass the bucket (0), or it might represent the time to begin and end the experiment (to lift the bucket and set it down at the end).

Step 12 Answers will vary. The advantage of having a systematic procedure is that everyone will arrive at the same model. Students may feel that the line through the Q-points does not capture the data as well as a line through two representative points.

SHARING IDEAS

Ask what Q-points and equations different groups found. Be sure students see that this method always yields the same lines.

Have groups share their ideas about the questions in Steps 11 and 12. Step 12 foreshadows Lesson 4.7. **[Ask]** "Why are the Q-points better than extreme points?" [Points at the extremes of data tend to be less reliable because measurements and instruments are often least accurate there. Also, in the real world, some relationships have nonlinear end-behavior, such as a rubber band stretched to its breaking point.]

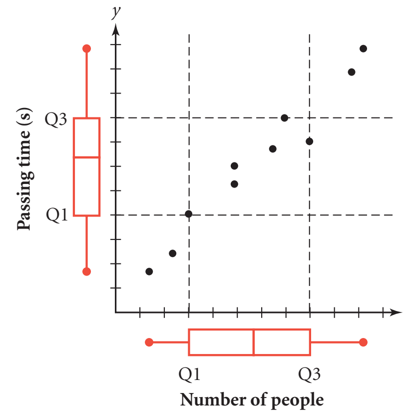
Ask when Q-points will be actual data points. Even if the quartiles are points in the one-variable data, the pairs of quartiles might not be points in the two-variable data.

Step 4 Let x represent the number of people, and let y represent time in seconds. Plot your data on graph paper.

Step 5 List the five-number summary for the x-values and the five-number summary for the y-values.

Step 6 What are the first-quartile (Q1) and third-quartile (Q3) values for the x-values in your data set? What are the Q1- and Q3-values for the y-values in your data set?

Step 7 On your graph, draw a horizontal box plot just below the x-axis using the five-number summary for the x-values. Draw a vertical box plot next to the y-axis using the five-number summary for the y-values. A sample graph is shown. Your data and graph will look different based on the data that you collect.



Step 8 Q-points may not be data points. Many factors influence this, including the number of data points and the strength of the linear relationship (how close the points are to a line). However, everyone should get the same Q-points.

Step 8 Draw vertical lines from the Q1- and Q3-values on the x-axis box plot into the graph. Draw horizontal lines from the Q1- and Q3-values on the y-axis box plot into the graph. These lines should form a rectangle in the plot. The vertices of this rectangle are called **Q-points.** Do the Q-points have to be actual data points? Why or why not? Will everyone get the same Q-points?

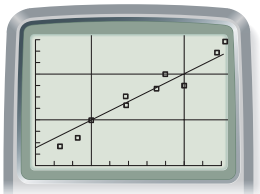
Step 9 Draw the diagonal of this rectangle that shows the direction of the data. Extend this diagonal through the plot. Is the line a good fit for the data? Are any of the original data points on your line? If so, which ones?

Step 10 Find the coordinates of the two Q-points the line goes through and write a point-slope equation for the line.

Step 11 What are the real-world meanings of the slope and y-intercept of this model?

Step 12 What are the advantages and disadvantages of having a systematic procedure for finding a model for data?

Step 13 Use your calculator to plot the data points, draw the vertical and horizontal lines, and plot a line of fit found by this method. [▶ See **Calculator Note 4B** for help on using the draw menu.◀]



LESSON OBJECTIVES

- Use quartiles to find an equation to fit a set of data
- Develop a strategy for agreeing on one equation for a set of data
- Review five-number summaries and box plots

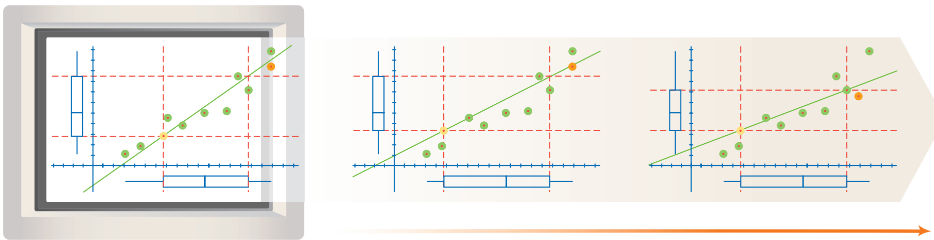
NCTM STANDARDS

	CONTENT		PROCESS
✓	Number	✓	Problem Solving
✓	Algebra	✓	Reasoning
	Geometry	✓	Communication
	Measurement	✓	Connections
✓	Data/Probability	✓	Representation

The method of finding a line of fit based on Q-points is more direct than the methods you used in Lessons 4.2 and 4.5. It is more systematic, too, because everyone will get the same points and the points themselves relate to measures of center in the upper and lower halves of the data set.

keymath.com/DA

[▶ For a **Dynamic Algebra Exploration** that investigates how moving one data point affects box plots and Q-points, see www.keymath.com/DA. ◀]



These students are collecting water samples. Their samples can be analyzed for many things, including dissolved oxygen.

EXAMPLE

The table lists the concentration of dissolved oxygen (DO) in parts per million at various temperatures in degrees Celsius from a sample of lake water. Find a line of fit based on Q-points for the data, and use it to predict the temperature for water with only 4 ppm dissolved oxygen.

Dissolved Oxygen

Temperature (°C) x	DO (ppm) y	Temperature (°C) x	DO (ppm) y
17	8	8	14
16	10	8	14
15	9	7	13
13	11	6	16
13	11	6	16
11	14	5	15
10	14	4	17
9	13		

Assessing Progress

From students' work on the investigation and their contributions to Sharing, you can assess their skill at recording data systematically, plotting points, finding five-number summaries, drawing box plots, and writing linear equations in point-slope form.

EXAMPLE

This example is good for students who didn't understand the investigation very well. In Lesson 4.5, Exercise 6, students found the equation of a line of fit for these data by guessing and adjusting. Now they will use the standardized method to find the equation of the line of fit based on Q-points. **[Ask]** "Why is the slope negative?" [As a point moves along the line from left to right, its y-value decreases while its x-value increases. Hence either the numerator or the denominator of the slope will be negative.]

The Fathom demonstration More on Modeling can be used to replace this example.

Closing the Lesson

As needed, say that one way to standardize the choice of two points through which a line of fit passes is to use the first and third quartiles for each data variable.

Solution

The five-number summaries are

For temperature (x-values): 4, 6, 9, 13, 17

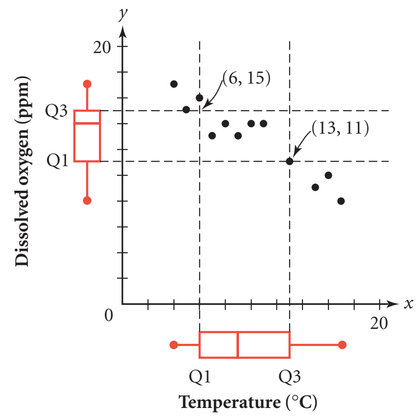
For dissolved oxygen (y-values): 8, 11, 14, 15, 17

The first-quartile and third-quartile values are

For the x-values: Q1 = 6, Q3 = 13

For the y-values: Q1 = 11, Q3 = 15

A sketch of the scatter plot shows that the appropriate Q-points are (6, 15) and (13, 11). Why are these the correct points, rather than (6, 11) and (13, 15)? Note that (6, 15) is not actually one of the data points but (13, 11) is.



Calculating the slope between these two points, you get

$$b = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(11 - 15)}{(13 - 6)} = \frac{-4}{7} \approx -0.57$$

This means that if the temperature *rises* 1°C, the dissolved oxygen concentration *decreases* by 0.57 ppm. It also means that if the temperature *drops* 1°C, the dissolved oxygen concentration *increases* by 0.57 ppm.

Using the slope −0.57 and the coordinates of the point (6, 15) in the point-slope form, $y = y_1 + b(x - x_1)$, gives

$$y = 15 - 0.57(x - 6)$$

To find the temperature when the concentration of dissolved oxygen is 4 ppm, substitute 4 for y in the equation and solve for x.

$y = 15 - 0.57(x - 6)$	Original equation.
$4 = 15 - 0.57(x - 6)$	Substitute 4 for y.
$-11 = -0.57(x - 6)$	Subtraction property (subtract 15 from both sides).
$19.3 \approx x - 6$	Division property (divide both sides by −0.57).
$25.3 \approx x$	Addition property (add 6 to both sides).

At about 25°C, the water will have about 4 ppm dissolved oxygen.

EXERCISES

Practice Your Skills

1. APPLICATION This table shows that the traveling distances between some cities depend on how you travel.

Traveling Distances

From	To	Flying distance (mi)	Driving distance (mi)
Detroit, MI	Memphis, TN	623	756
St. Louis, MO	Minneapolis, MN	466	559
Dallas, TX	San Francisco, CA	1483	1765
Seattle, WA	Los Angeles, CA	959	1150
Washington, DC	Pittsburgh, PA	192	241
Philadelphia, PA	Indianapolis, IN	585	647
New Orleans, LA	Chicago, IL	833	947
Cleveland, OH	New York, NY	405	514
Birmingham, AL	Boston, MA	1052	1194
Denver, CO	Buffalo, NY	1370	1991
Kansas City, MO	Omaha, NE	166	204

[Data sets: FLYDS, DRVDS]

a. What are the five-number summary values of the flying distances? @ 166, 405, 623, 1052, 1483

b. What are the five-number summary values of the driving distances? @ 204, 514, 756, 1194, 1991

c. Plot the data points. Let x represent flying distance in miles, and let y represent driving distance in miles. @

d. Will the slope of the line through these points be positive or negative? Explain your reasoning. @ The slope will be positive because as the flying distance increases so does the driving distance.

e. Use the five-number summary values to draw a rectangle on the graph of the data. Name the two Q-points you should use for your line of fit. @

f. Find the equation of the line and graph the line with your data points.

g. The flying distance from Louisville, Kentucky, to Miami, Florida, is 919 miles. Predict the driving distance from Louisville to Miami. @ approximately 1054 mi

h. The driving distance from Phoenix, Arizona, to Salt Lake City, Utah, is 651 miles. Predict the flying distance from Phoenix to Salt Lake City. approximately 535 or 536 mi

BUILDING UNDERSTANDING

Students get practice in using Q-points to develop lines of fit.

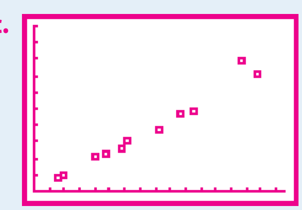
ASSIGNING HOMEWORK

Essential	1–4, 7 or 8
Performance assessment	7, 8
Portfolio	4
Journal	3, 6, 12
Group	5, 9
Review	10–12

Helping with the Exercises

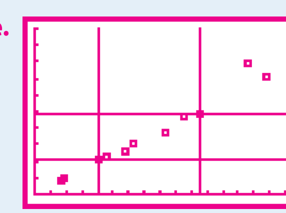
Exercise 1 Either column of data may be considered input or output. Choosing the driving distance as input would give different equations. Different procedures for rounding will also lead to a variety of answers.

1c.



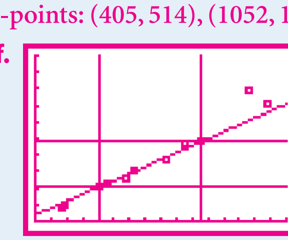
[0, 1650, 100, 0, 2500, 250]

1e.



Q-points: (405, 514), (1052, 1194)

1f.



The slope is approximately 1.05; $y = 1194 + 1.05(x - 1052)$ or $y = 514 + 1.05(x - 405)$.

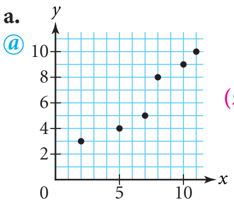
2. **APPLICATION** Let x represent total fat in grams, and let y represent saturated fat in grams. Use the model $y = 10 + 0.5(x - 28)$ to predict

a. The number of saturated fat grams for a hamburger with a total of 32 grams of fat. 12 g saturated fat

b. The total number of fat grams for a hamburger with 15 grams of saturated fat. 38 g total fat

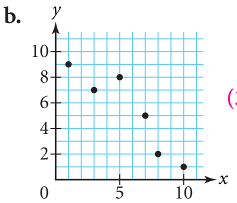
3. Give the coordinates of the Q-points for each data set.

a. ⓐ



(5, 4), (10, 9)

b.



(3, 8), (8, 2)

Reason and Apply

4. The table gives the winning times for the Olympic men's 10,000-meter run.

a. Define variables and find the line of fit based on Q-points for the data.

b. Plot the data points and graph the equation of the model to verify that it is a good fit. ⓐ

c. What is the real-world meaning of the slope?

d. Kenenisa Bekele of Ethiopia won the 10,000-meter race in the 2004 Olympic Games. Compare his actual winning time of 27.08 minutes with the winning time predicted by your model.

e. Could you use this model to predict the winning time 100 years from now? Explain why or why not.

Men's 10,000-meter Run

Year	Champion	Time (min)
1952	Emil Zatopek, Czechoslovakia	29.28
1956	Vladimir Kuts, USSR	28.76
1960	Pyotr Bolotnikov, USSR	28.54
1964	Billy Mills, United States	28.41
1968	Naftali Temu, Kenya	29.46
1972	Lasse Viren, Finland	27.64
1976	Lasse Viren, Finland	27.67
1980	Miruts Yifter, Ethiopia	27.71
1984	Alberto Cova, Italy	27.79
1988	Brahim Boutaib, Morocco	27.36
1992	Khalid Skah, Morocco	27.78
1996	Haile Gebrselassie, Ethiopia	27.12
2000	Haile Gebrselassie, Ethiopia	27.30

(International Olympic Committee, in *The World Almanac and Book of Facts 2004,* p. 866) [Data sets: **RUNYR, RUNTM**]

5. Create a data set that has Q-points at (4, 28) and (12, 47) so that only one of those two points is actually part of the data set. ⓗ

6. Which linear equation below best fits the data at right? Explain your reasoning.

i. $y = 1.3 + 0.18(x - 6)$

ii. $y = 2.2 + 0.18(x - 6)$

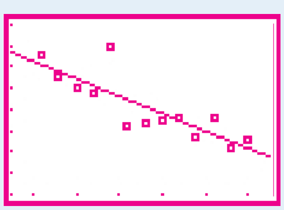
iii. $y = 1.3 - 0.18(x - 6)$

iv. $y = 2.2 - 0.18(x - 6)$

Time (s) x	Distance from motion sensor (m) y
2	2.8
6	2.2
8	1.7
9	1.5
11	1.3
14	0.9

4a. Let x represent years, and let y represent winning time in minutes. The five-number summary for x is 1952, 1962, 1976, 1990, 2000. The five-number summary for y is 27.12, 27.5, 27.78, 28.65, 29.46. The Q-points are (1962, 28.65) and (1990, 27.5). The slope of the line through these two points is about -0.0411, so the possible equations are $y = 28.65 - 0.0411(x - 1962)$ and $y = 27.5 - 0.0411(x - 1990)$.

4b.



[1945, 2005, 10, 26, 30, 0.5]

4c. The slope, -0.0411, means that the winning time decreases by an average of 0.0411 min (2.47 s) each year.

4d. The prediction is 26.92 min, which is 0.16 min (9.6 s) less than the actual winning time.

Exercise 4e This question shows the limitations of the model for making long-range predictions.

4e. Answers will vary. However, there is a physical limit to how fast a runner can run. Eventually, the times will have to level off.

5. Answers will vary. One example is {(2, 22), (4, 30), (6, 28), (8, 35), (10, 42), (12, 47), (14, 53)}.

Exercise 6 Encourage lots of approaches here. Some students may find the Q-points and see which line goes through them. Others may count the number of data points on each side of a line or measure the vertical distance from data points to the line.

6. Reasons will vary. The data pattern has a negative slope, and the Q-points that lie on the line are (11, 1.3) and (6, 2.2). The point (6, 1.3) is not one of the Q-points used to draw the line, so equation iv is the correct equation.

7. At 2:00 P.M., elevator A passes the second floor of the Empire State Building going up. The table shows the floors and the times in seconds after 2:00.

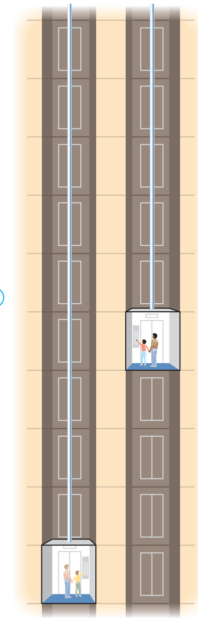
Floor x	2	4	6	8	10	12	14
Time after 2:00 (s) y	0	1.3	2.5	3.8	5	6.3	7.5

a. What is the line of fit based on Q-points for the data? @

b. Give a real-world meaning for the slope. @ The elevator is rising at a rate of 0.625 s per floor.

c. About what time will this elevator pass the 60th floor if it makes no stops? @

d. Where will this elevator be at 2:00:45 if it makes no stops? @ almost at the 74th floor



8. At 2:00 P.M., elevator B passes the 94th floor of the same building going down. The table shows the floors and the times in seconds after 2:00.

Floor x	94	92	90	88	86	84	80
Time after 2:00 (s) y	0	1.3	2.5	3.8	5	6.3	8.6

a. What is the line of fit based on Q-points for the data?

b. Give a real-world meaning of the slope. The elevator is moving down at 0.625 s per floor.

c. About what time will this elevator pass the 10th floor if it makes no stops?

d. Where will this elevator be at 2:00:34 if it makes no stops? between the 39th and 40th floors

9. Think about the elevators in Exercises 7 and 8.

a. Estimate when elevator A will pass elevator B if neither makes any stops. Estimates will vary.

b. Calculate the actual time. At 28.8 s, or at about 2:00:29, the elevators will pass at the 48th floor.

Review

3.2 **10.** A car is traveling from Sioux Falls, South Dakota, to Mt. Rushmore, which is near Rapid City, South Dakota. The car is traveling about 54 mi/h, and it is about 370 mi from Sioux Falls to Mt. Rushmore.

a. Write a recursive routine to create a table of values in the form (*time, distance from Mt. Rushmore*) for the relationship from 0 to 6 h. @

b. Graph a scatter plot using 1 h time intervals.

c. Draw a line through the points of your scatter plot. What is the real-world meaning of this line? What does the line represent that the points alone do not?

d. What is the slope of the line? What is the real-world meaning of the slope?

e. When will the car be at the Wall Drug Store, which is 80 mi from Mt. Rushmore? Explain how you know.

f. When will the car arrive at Mt. Rushmore? Explain how you know.

Wall Drug is a landmark in South Dakota. The store's fame began during the Great Depression, when it offered free ice water to travelers.

Exercise 7 In this exercise, as in the investigation, time is more reasonably an output than an input variable. **[Alert]** Some students may need help interpreting 2:00:45 as 45 s after 2:00.

7a. $y = 1.3 + 0.625(x - 4)$ or $y = 6.3 + 0.625(x - 12)$

7c. 36.3 s after 2:00, or at approximately 2:00:36

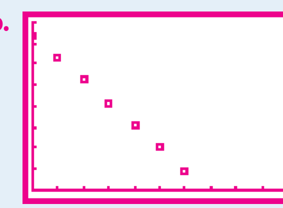
8a. $y = 1.3 - 0.625(x - 92)$ or $y = 6.3 - 0.625(x - 84)$

8c. after 52.55 s, or at approximately 2:00:53

10a. Start with 370, then use the rule Ans − 54.

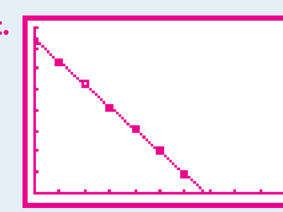
Time (h)	Distance from Mt. Rushmore (mi)
0	370
1	316
2	262
3	208
4	154
5	100
6	46

10b.



[0, 10, 1, 0, 400, 50]

10c.



The line represents the distance remaining at any time during the trip. With the line, you can see how far away from Mt. Rushmore you are at any time, instead of just at the top of each hour.

10d. −54; the real-world meaning of the slope is that your distance from Mt. Rushmore decreases by 54 mi each hour.

10e. Answers will vary. The car will reach the Wall Drug Store in the first half hour of the fifth hour of the trip. You can see this on the graph if you look at the line where it has a y-value of about 80.

10f. The car will reach Mt. Rushmore after almost 7 h of travel. You can see this on the graph or in the table, because after 7 h, the car would have gone 8 mi too far.

11. The size and cost are almost directly proportional; the 4 oz bottle costs \$0.22/oz, the 7.5 oz bottle costs \$0.22/oz, and the 18 oz bottle costs \$0.2217/oz. If you change the price of the 18 oz bottle to \$3.96, then it also will cost exactly \$0.22/oz.

Exercise 12 Have students read and critique each other's e-mails. A good technique is to choose a few papers, delete the students' names, make overheads, and have each class critique the explanations from another class.

2.4 **11.** A 4 oz bottle of mustard costs \$0.88, a 7.5 oz bottle costs \$1.65, and an 18 oz bottle costs \$3.99. Is the size of the mustard bottle directly proportional to the price? If so, show how you know. If not, suggest the change of one or two prices so that they will be directly proportional. ⓗ

4.4 **12.** Imagine that a classmate has been out of school for the past few days with the flu. Write him or her an e-mail describing how to convert an equation such as $y = 4 + 2(x - 3)$ from point-slope form to slope-intercept form. Be sure to include examples and explanations. End your note by telling your classmate how to find out if the two equations are equivalent. Answers will vary.

project

STATE OF THE STATES

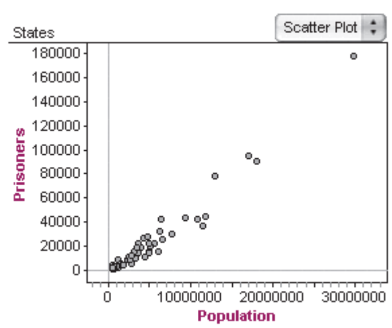
Many characteristics of a state vary with the size of the state's population. Some of these relationships are linear. The more people who live in a state, the more houses, cars, schools, and prisoners there are. A lot of data about the states is available on the Internet. You can link to a useful site through www.keymath.com/DA .

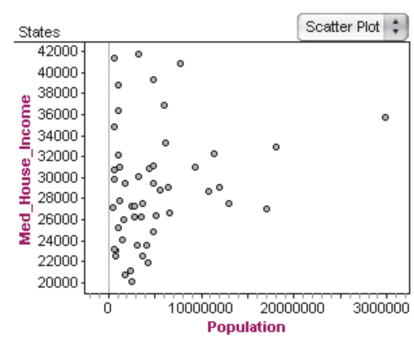
Here are two scatter plots that show a comparison of the population of a state to two different characteristics of the state—number of prisoners and median household income. Which scatter plot shows a linear pattern?

Fathom

Fathom comes with many data sets that contain information about the states, and you can easily download more information from websites.

Plot quartiles and use Fathom's movable line to find the slope.





Investigate various pairs of states' characteristics that you think might be related.

Your project should include

- Several scatter plots, investigating relationships between various pairs of characteristics for states.
- Lines of fit for your plotted data, their slopes and intercepts along with their real-world meanings (that is, if there appears to be a linear relationship).
- Explanations of why some relationships do not appear linear.

Supporting the project

MOTIVATION

The project gives students a chance to study data sets and decide how close to linear they are. **[Ask]** "What characteristics of a state are related to the population?" [One answer: The number of prisoners has a linear relationship to the population.]

OUTCOMES

- The report includes several scatter plots of data.
- Lines of fit are graphed on the scatter plots.
- Slopes and intercepts of the lines of fit are included.
- There's a discussion of why the data are linear or nonlinear.
  - The student explains how the lines of fit were determined.
  - The student assesses the goodness of the fit.
  - Predictions are made on the basis of the lines of fit.

LESSON 4.7

Applications of Modeling

In Lesson 4.6, you learned a systematic method, using quartile values, to find a line of fit for data points that appear to have a linear pattern. In this lesson, you'll contrast that method with ways you've used before and evaluate your results.

Investigation
What's My Line?

You will need

- graph paper
- a strand of spaghetti

This table shows data that Edwin Hubble used in 1929 to formulate Hubble's Law. The table includes the distance from Earth to known nebulae (clouds of gas or dust) measured in megaparsecs (Mpc, about 3.258×10^6 light-years), and the speed at which each nebula is moving away from or toward Earth. In this investigation, you'll create three linear models, you'll analyze your models, and you'll use them to make a prediction.

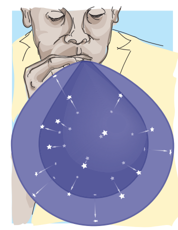
Science CONNECTION

In 1929, American astronomer Edwin Hubble (1889–1953) formulated Hubble's Law, which describes the rate at which galaxies move away from each other. This law led to the concept of an expanding universe, and, working back in time, it also provides a basis for the big bang theory. For more information on Hubble's Law and the big bang theory, see www.keymath.com/DA .

Distance and Speed of Nebulae

Distance (Mpc)	Speed (km/s)	Distance (Mpc)	Speed (km/s)
0.032	170	0.9	650
0.034	290	0.9	150
0.214	−130	0.9	500
0.263	−70	1.0	920
0.275	−185	1.1	450
0.275	−220	1.1	500
0.45	200	1.4	500
0.5	290	1.7	960
0.5	270	2.0	500
0.63	200	2.0	850
0.8	300	2.0	800
0.9	−30	2.0	1090

(Edwin Hubble, in *Proceedings of the National Academy of Sciences,* Volume 15, Number 3)
[Data sets: **GXYDS, GXYSP**]



The expanding-universe theory can be illustrated by placing dots on a balloon and inflating it. The dots represent galaxies. If you imagine standing in one galaxy, you'll see that galaxies farther from you move away at a faster rate than galaxies that are closer.

First, you'll find a line of fit using an "eyeballing" method. Remember that the object of a linear model is to summarize or generalize the data.

Step 1 Plot the data on graph paper. Lay a piece of spaghetti on the plot so that it crosses the y-axis and follows the direction of the data. Try to focus not on the points themselves, but on the general direction of the "cloud" of points.

Step 2 Estimate the y-intercept. Locate a point with convenient coordinates along the strand. Use this information to write the equation of the line.

Step 2 Answers will vary. Students will need to use the y-intercept and the second point to find the slope—remind them that the x-coordinate for the y-intercept is 0.

LESSON 4.7

PLANNING

LESSON OUTLINE

One day:

30 min	Investigation
5 min	Sharing
5 min	Closing
10 min	Exercises

MATERIALS

- graph paper
- uncooked spaghetti

TEACHING

In this lesson students practice their skills of finding and evaluating lines of fit. They compare the results of the different methods when applied to the same data set.

Guiding the Investigation

One Step
Ask students to find lines of fit for the data in the table in as many ways as possible and to decide which line fits best. Encourage a variety of approaches, at least reviewing all methods seen so far.

You might want to use Fathom for this investigation if it's available. Students will need the Fathom skills shown in the Fathom demonstrations for Lessons 4.5 and 4.6.

Step 1 How to assign input and output variables may not be clear to students. Neither variable is obviously the independent variable. In this case most mathematicians would use the horizontal axis for the first variable listed, *distance.*

See page 262 for answers to Step 1.

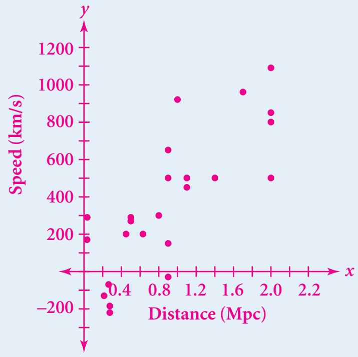
NCTM STANDARDS

CONTENT		PROCESS	
✓	Number	✓	Problem Solving
✓	Algebra	✓	Reasoning
	Geometry	✓	Communication
	Measurement	✓	Connections
✓	Data/Probability	✓	Representation

LESSON OBJECTIVES

- Review various approaches to finding lines to fit sets of data
- Evaluate the results of those approaches

Step 1



Step 2 Because students have found the y-intercept, they will probably want to use the intercept form for the equation. Try to be sure all students know how to do that.

Step 5 As needed, suggest that students draw horizontal and vertical lines to help find the Q-points.

Step 6 Although no particular form of the equation is requested, students will find it easiest to follow the precedent of Step 4 and find the point-slope form.

Step 8 Answers will vary. Using the equation from Step 6, the y-intercept is the speed at which a galaxy would be moving away from Earth if the galaxy were located at the same place as Earth.

Step 10 As you change the y-intercept, the y-coordinate of every point on the line increases by the same amount. For example, if the y-intercept changes from -10 to 0, the point $(1, 458)$ moves to $(1, 468)$.

Step 11 A distance "far from most of the given points" might be 3.

Steps 3 and 4 Answers will vary.

Next, you'll find a line of fit by choosing "representative" data points.

Step 3 Make a scatter plot of the data on your calculator. Choose two data points that you think show the direction of the data.

Step 4 Use the two points to write a linear equation in point-slope form.

Next, you'll find a line of fit using Q-points.

Step 5 Use your calculator to get the five-number summaries for the x- and y-values. Draw a rectangle using the first- and third-quartile values for the x-values and the first- and third-quartile values for the y-values. Name the Q-points you should use for the data. x-values: 0.032, 0.3625, 0.9, 1.25, 2.0; y-values: -220, 160, 295, 575, 1090; Q-points: $(0.3625, 160)$, $(1.25, 575)$

Step 6 Write the equation of the line of fit you can draw through your selected Q-points. Graph the equation to verify that it is the diagonal of the rectangle you drew on the plot. $y = 160 + 468(x - 0.3625)$ or $y = 575 + 468(x - 1.25)$

Step 7 Answers will vary. The slopes all represent the rate at which speed (in km/s) increases for each increase of one megaparsec.

Step 11 A small change in the slope will have a magnifying change for points far out on the line. For example, if the equation $y = 1x$ is changed to $y = 5x$, the point $(0.1, 0.1)$ only changes to $(0.1, 0.5)$, but the point $(100, 100)$ changes to $(100, 500)$.

Finally, you'll compare the lines and their characteristics and decide which method has given you the best-fitting line.

Step 7 Compare the slopes of all three lines for each table. Do all these numbers have the same real-world meaning? If so, what is it?

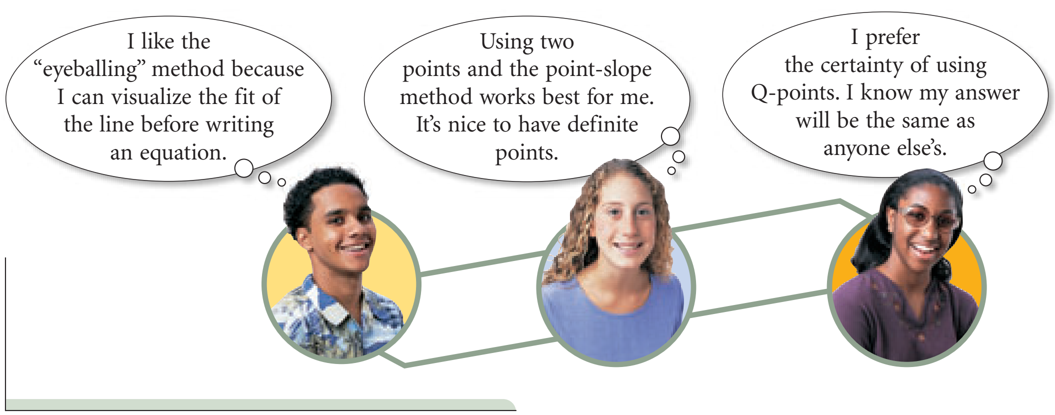
Step 8 Compare the y-intercepts of all three lines for each table. Do they all have the same real-world meaning? If so, what is it?

Step 9 What distance would you expect for a galaxy that is moving away from Earth at a rate of 750 km/s? Show how to find this value symbolically. 1.6 Mpc

Step 10 What is the effect of a small change in the y-intercept when you use the model to predict a value in the middle of the data set?

Step 11 What is the effect of a small change in the slope when you try to predict a y-value far from most of the given points?

Step 12 Discuss the pros and cons of each procedure you used to find a line of fit. Which method do you like best and why? Answers will vary.



SHARING IDEAS

After Step 6, have several groups graph their lines, using a single overhead transparency for each table of data and a different color for each group. You might lead a full-class discussion of Steps 7–12, or ask students to return to their groups. In a class discussion, be especially careful to treat all answers to questions equally, asking the class to critique them, whether or not you agree with them. (If some students seem uncomfortable participating in the class discussion, adjust your plans and return to groups.)

Ask if a line can be a reliable predictor if it's not the best fit. Elicit the idea that the quality of fit is determined by the accuracy of the prediction, so it depends on the situation.

If your class is ready, you might introduce the median-median method for finding a line of fit. (See page 214B.) Students can check hand calculations with a calculator. Ask if this line is a better model of the data.

As you study more about finding models for data, you will also learn more about methods you can use to tell how well a model fits data. In this course the emphasis will be on finding a reasonable model—even though it may not be the best-fitting model—so that you can use it to make reliable predictions.

EXERCISES

You will need your graphing calculator for Exercise **6.**

Practice Your Skills

1. The equation of a line in point-slope form is $y = 6 - 3(x - 6)$.

a. Name the point on this line that was used to write the equation. $(6, 6)$

b. Name the point on this line with an x-coordinate of 5. $(5, 9)$

c. Using the point you named in 1b, write another equation of the line in point-slope form. $y = 9 - 3(x - 5)$

d. Write the equation of the line in intercept form. $y = 24 - 3x$

e. Find the coordinates of the x-intercept. $(8, 0)$

2. Solve each equation symbolically for x. Use another method to verify your solution.

a. $3(x - 5) + 14 = 29$ @ $x = 10$ **b.** $\frac{8 - 13}{x + 5} = 2$ $x = -7.5$

c. $\frac{2(3 - x)}{4} - 8 = -7.75$ $x = 2.5$ **d.** $11 + \frac{6(x + 5)}{9} = 42$ $x = 41.5$

3. Solve each equation for y.

a. $2x + 5y = 18$ @
$y = \frac{18 - 2x}{5}$, or $y = 3.6 - 0.4x$

b. $5x - 2y = -12$
$y = \frac{-12 - 5x}{-2}$, or $y = 6 + 2.5x$

Reason and Apply

4. **APPLICATION** This table shows winning distances for the Olympic men's discus throw.

Men's Discus

Year	Champion	Distance (m)
1952	Sim Iness, United States	55.03
1956	Al Oerter, United States	56.36
1960	Al Oerter, United States	59.18
1964	Al Oerter, United States	61.00
1968	Al Oerter, United States	64.78
1972	Ludvik Danek, Czechoslovakia	64.40
1976	Mac Wilkins, United States	67.50
1980	Viktor Rashchupkin, USSR	66.64
1984	Rolf Danneberg, West Germany	66.60
1988	Jürgen Schult, East Germany	68.82
1992	Romas Ubartas, Lithuania	65.12
1996	Lars Riedel, Germany	69.40
2000	Virgilijus Alekna, Lithuania	69.30
2004	Virgilijus Alekna, Lithuania	69.89

(International Olympic Committee, in *The World Almanac and Book of Facts 2004,* p. 867)

a. Define variables and find the line of fit based on Q-points for this data set. Give the real-world meanings of the slope and the y-intercept. @

b. In 1912, Armas Taipale of Finland threw the discus 45.21 m. What value does your model predict for that year? What is the difference in the two values?

c. According to your model, what year might you expect the winning distance to pass 80 m? Show how to find this value symbolically.

Assessing Progress

Watch for students' ability to plot points, find slopes and y-intercepts, write equations of lines given the y-intercept and a slope, write equations given points and slopes, find lines of fit using Q-points, figure out the real-world meaning of a slope or y-intercept, and find the value of an expression at a point.

Closing the Lesson

A variety of approaches yield lines to model a set of data. The best of these will be a good predictor of data points not already found.

BUILDING UNDERSTANDING

Students practice making predictions from lines of fit found using Q-points.

ASSIGNING HOMEWORK

Essential	1, 4
Performance assessment	5
Portfolio	4
Journal	8
Group	2
Review	1–3, 6–8

4a. Let x represent years, and let y represent distance in meters. The Q-points are (1964, 61.00) and (1992, 68.82). The slope of the line through these points is about 0.28, so the equation is $y = 61.00 + 0.28(x - 1964)$ or $y = 68.82 + 0.28(x - 1992)$. The slope, 0.28, means that the winning distance increases an average of 0.28 m, or 28 cm, each year. The y-intercept, -489 m, is meaningless in this situation because it would indicate that a negative distance was the winning distance in year 0. The model cannot predict that far out from the data range.

4b. 46.44 m using $y = 61.00 + 0.28(x - 1964)$; the predicted distance is 1.23 m more than the actual distance.

4c. 2032 using $80 = 61.00 + 0.28(x - 1964)$

Helping with the Exercises

Exercise 5 **[Alert]** Some students may not realize that columns 1 and 3 are extraneous. Either column 2 or 4 can be used for input data and the other for output.

5a. Let x represent distance from Los Angeles in miles, and let y represent elapsed time from Seattle in minutes; $y = 1439 - 1.51(x - 411.5)$ or $y = 273 - 1.51(x - 1181.5)$; the slope means the distance from Los Angeles decreases by 1 mi each 1.51 min.

5b. approximately 1758, or 29 h 18 min, by the first equation or approximately 1755 min, or 29 h 15 min, by the second equation

5c. approximately 967 mi by the first equation or 965 mi by the second equation

5. **APPLICATION** The table shows the timetable for the Coast Starlight train from Seattle to Los Angeles.

Coast Starlight

Location	Distance from Los Angeles (mi)	Arrival time	Elapsed time from Seattle (min)
Kelso, WA	1252	12:48	168
Vancouver, WA	1213	13:29	209
Salem, OR	1150	15:37	337
Eugene, OR	1079	17:10	430
Sacramento, CA	552	6:35	1205
Emeryville, CA	468	8:10	1330
Salinas, CA	355	11:48	1548
Santa Barbara, CA	103	18:17	1937

Before 1971, when Amtrak created the Coast Starlight, passengers had to ride three different trains to go from Seattle to Los Angeles.

a. Define variables and give the line of fit based on Q-points for this data set. Give the real-world meaning of the slope.

b. While riding the train, you pass a sign that says you are 200 mi from Los Angeles. What length of time does your model predict you have traveled?

c. The train comes to a stop after 10 h (600 min). According to your model, how far are you from Los Angeles? Show how to find this value symbolically.

Review

6. In Chapter 3 you worked with problems involving rate, often involving the equation $d = rt$. Here is another kind of **rate problem.**

Ellen and Eric meet on Saturday to train for a marathon. They live 7 miles apart and meet at the high-school track that is between their two homes. Ellen leaves at 8:00 A.M. and jogs south toward the school at 4 mi/h. Eric waits until 8:30 A.M. and jogs north toward the school at 6 mi/h. The two friends arrive at the school at exactly the same time. How much time did each person jog?

To solve this problem, let t represent Ellen's time in hours. Because Eric left a half hour after Ellen, but arrived at the same time, he jogged for a half hour less. So let $t - \frac{1}{2}$ represent Eric's time in hours. You might now fill out a table like this to get expressions for distance. (Remember that *distance* = *rate* · *time*.)

	rate (mi/h)	*time* (h)	*distance* (mi)
Ellen	4	t	$4 \cdot t$
Eric	6	$t - \frac{1}{2}$	$6 \cdot \left(t - \frac{1}{2}\right)$
Combined			7

a. Write an equation that states that Ellen's distance and Eric's distance combine to 7 miles. $4t + 6\left(t - \frac{1}{2}\right) = 7$

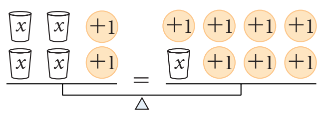
b. Solve the equation from 6a, check your answer, and state the solution.

c. Solve this problem using a similar procedure: A propeller airplane and a jet airplane leave the same airport at the same time, and both go in the same direction. The jet airplane's velocity is five times the propeller airplane's velocity. After 2.25 h, the jet airplane is 1170 km ahead of the propeller plane. What is the velocity of each plane in kilometers per hour? propeller airplane: 130 km/h; jet airplane: 650 km/h

6b. $t = 1$; Ellen jogged for 1 hour and Eric jogged for $\frac{1}{2}$ hour.

2.3 **7.** A sample labeled "50 grains" weighs 3.24 grams on a balance. What is the conversion factor for grams to grains? 15.4321 grains per gram

3.6 **8.** Write the equation represented by this balance. Then solve the equation for x using the balancing method.



$4x + 2 = x + 7$	Original equation.
$4x - x + 2 = x - x + 7$	Subtract x from both sides.
$3x + 2 = 7$	Combine like terms.
$3x + 2 - 2 = 7 - 2$	Subtract 2 from both sides.
$3x = 5$	Combine like terms.
$\frac{3x}{3} = \frac{5}{3}$	Divide both sides by 3.
$x = \frac{5}{3}$, or $1.\overline{6}$	Reduce.

IMPROVING YOUR REASONING SKILLS

Not all data sets form a linear pattern. Here is a set that doesn't. It relates speed and time for the same car trip made by several drivers. Plot the data and see if you recognize the shape. Once you do, write an equation whose graph shows this shape. Then adjust it, if necessary, to better show the shape of the data. Use your equation to predict how much time a driver who averages 45 mi/h would need for the trip and the average speed that would give a time of 70 min.

Average Speed and Time for the Same Trip

Average Speed (mi/h) x	Time (min) y	Average Speed (mi/h) x	Time (min) y
25	144	45	
26	137.6	50	72
30	120		70
34	106.1	55	65.5
36	99.7	56.5	63.7
37.4	96.4	60	59.8
40.5	89.3	62	58
42.2	85.4	65	55.5

IMPROVING REASONING SKILLS

This relationship is an inverse variation. Students can find the product xy for each speed and average them to get the constant of variation, approximately 3600. Using this approach, the curve of fit is $y = \frac{3600}{x}$.

According to this model, at an average speed of 45 mi/h, the trip would take approximately 80 min. To take 70 min, the average speed would have to be approximately 51.4 mi/h.

[20, 70, 5, 50, 150, 10]

LESSON
4.8

PLANNING

LESSON OUTLINE

First day:

5 min Introduction

35 min Activity (Steps 1–7)

10 min Quiz or Review exercises

Second day:

25 min Activity (Steps 8 and 9)

15 min Sharing

10 min Review exercises

MATERIALS

- toy figures
- identical rubber bands (about 300)
- tape measures, metersticks, or yardsticks
- video camera, *optional*
- Lab Report (W from Chapter 1), *optional*

TEACHING

Besides predicting the output value of a linear equation for a given input value, you often need to determine the input value for a given output value. Students do that in this activity.

Guiding the Activity

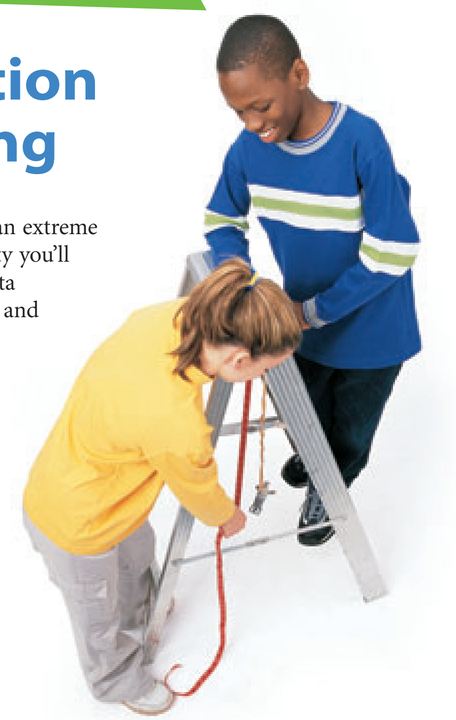
A compact toy like a 10 in. action figure, stuffed animal, or beanbag works best. You can also use a small water balloon or a plastic water bottle with a little water in it. Size 32 rubber bands work well.

LESSON
4.8

Activity Day

Data Collection and Modeling

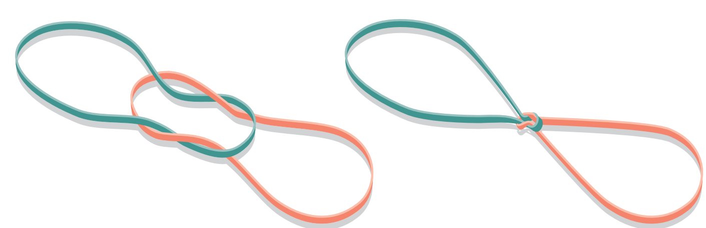
Here's your chance to take part in an extreme sport without the risk! In this activity you'll set up a bungee jump and collect data relating the distance a "jumper" falls and the number of rubber bands in the bungee cord. Then you'll model the data with an equation. Next you'll use your model to find the number of rubber bands you'd need in the cord for a near miss from a specific height.



Activity
The Toyland Bungee Jump

You will need

- a toy to serve as "jumper"
- a supply of equal-sized rubber bands
- a tape measure



Step 1 Make a bungee cord by attaching two rubber bands to your "jumper." (You may first need to make a harness by twisting a rubber band around the toy.)

Step 2 Place your jumper on the edge of a table or another surface while holding the end of the bungee cord. Then let your jumper fall from the table. Use your tape measure to measure the maximum distance the jumper falls on the first plunge.

LESSON OBJECTIVES

- Review the process of finding equations to model data
- Learn about writing lab reports
- Learn about careful data collection and attention to procedure

NCTM STANDARDS

	CONTENT		PROCESS
	Number	✓	Problem Solving
✓	Algebra	✓	Reasoning
	Geometry	✓	Communication
✓	Measurement	✓	Connections
✓	Data/Probability	✓	Representation

Step 3 Repeat this jump several times and find a mean value for the distance. Record the number of rubber bands (2) and the mean distance the jumper falls in a table like this one.

Number of rubber bands	2	4	5	6
Distance fallen				

Step 4 Add one or two rubber bands to the bungee cord and repeat the experiment. Record this new information.

Step 5 Continue to make bungee cords of different lengths, and measure the distance your jumper falls until you have at least seven pairs of data. When using long cords, you may need to move to a higher place to measure the falls.

Step 6 Define variables and make a scatter plot of the information from your table.

Step 7 Find the equation of a line of fit for your data. You may use any procedure, but be able to justify why your equation is a reasonable fit.

Step 8 The test! Decide on a good location for all the groups to conduct final bungee jumps from a particular height. Use your equation to determine the number of rubber bands you need in the cord to give your jumper the greatest thrill—falling as close as possible to the ground without touching. When you have determined the number of rubber bands, make the bungee cord and wait your turn to test your prediction.

Step 9 Write a group report for this activity. Follow this outline to produce a neat, organized, thorough, and accurate report. Any reader of your report should not need to have watched the activity to know what is going on.

Report Outline

A. Overview	Tell what the investigation was about, its purpose or objective.
B. Data collection	Describe the data you collected and how you collected it.
C. Data table	Use labels and units.
D. Graph	Show all data points. Use labels and units. Show the line of fit.
E. Model	Define your variables and give the equation. Tell how you found this equation and why you used this method.
F. Calculations	Show how you decided how many rubber bands to use in the final jump.
G. Results	Describe what happened on the final jump.
H. Conclusion	What problems did you have? What worked really well? If you could repeat the whole experiment, what would you do to improve it?

Step 3 Students may want to proceed after one measurement. Ask them why they should take the mean of several measurements.

Step 8 Students can solve the equation by undoing or balancing. You'll want to guide them in selecting a location. Bleachers, a balcony, a stairwell, or a second-story window work well. You might advise students that they will have one opportunity to conduct their final jump. They should have their rubber bands in place before going to the test location.

If possible, set up a video camera to film the final drops. Focus on the area near the floor or ground. Then view the videotape in slow motion, one frame at a time, to see just how close to the ground the object comes. You can actually measure a distance on the television screen and then compare the various distances to determine the best bungee jump. Or, a student can hold a meterstick behind the drop location to be captured on the tape.

You might hand out the Lab Report Worksheet from Chapter 1 if you haven't had students write a lab report before.

SHARING IDEAS

You might ask the groups that made the best predictions to share with the class parts D–F of their report.

Assessing Progress
Assess students' ability to work together, make careful measurements, gather data systematically, find the mean of a data set, make scatter plots, find equations of lines of fit, and solve linear equations.

Closing the Lesson

As needed, point out that after finding the best line of fit, students had to solve a linear equation to predict the number of rubber bands for a given distance.

CHAPTER REVIEW 4

PLANNING

LESSON OUTLINE

One day:

10 min	Introduction
25 min	Exercises
15 min	Student self-assessment

REVIEWING

Direct attention to the Nutrition Facts data from the example in Lesson 4.2. Ask the class how to predict the saturated fat in a burger with 50 g total fat. Be sure students describe a variety of ways of finding a line of fit—eyeballing, finding a line through representative points, using Q-points. Don't be satisfied with a list. Ask for demonstrations of techniques, emphasizing how to find the slope of a line with slope triangles and how to find the point-slope form of an equation. Show how different equations might arise from different choices of points, and review the algebraic properties used to check the equivalence of the equations and to solve equations.

ASSIGNING HOMEWORK

If students complete the odd-numbered exercises on their own, they could do the even-numbered ones in groups.

CHAPTER 4 REVIEW

In Chapter 3, you learned how to write equations in intercept form, $y = a + bx$. In this chapter, you learned how to calculate **slope** using the slope formula, $b = \frac{y_2 - y_1}{x_2 - x_1}$. You also used the slope formula to derive another form for a linear equation—the **point-slope form.** The point-slope form, $y = y_1 + b(x - x_1)$, is the equation of a line through point (x_1, y_1) with slope b. You learned that this form is very useful in real-world situations when the starting value is not on the y-axis.

You investigated equivalent forms of expressions and equations using tables and graphs. You used the **distributive property** of multiplication over addition and the **commutative** and **associative** properties of addition and multiplication to write point-slope equations in intercept form.

You investigated several methods of finding a **line of fit,** and you discovered how to use the first and third quartiles from the five-number summaries of x- and y-values in a data set to write a linear model for data based on **Q-points.**

EXERCISES

You will need your graphing calculator for Exercises **3, 4,** and **9.**

@ Answers are provided for all exercises in this set.

1. The slope of the line between (2, 10) and $(x_2, 4)$ is -3. Find the value of x_2.

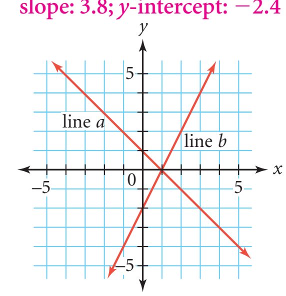
2. Give the slope and the y-intercept for each equation.

a. $y = -4 - 3x$
slope: -3; y-intercept: -4

b. $2x + 7 = y$
slope: 2; y-intercept: 7

c. $38x - 10y = 24$
slope: 3.8; y-intercept: -2.4

3. Line a and line b are shown on the graph at right. Name the slope and the y-intercept, and write the equation of each line. Check your equations by graphing on your calculator.



4. Write each equation in the form requested. Check your answers by graphing on your calculator.

a. Write $y = 13.6(x - 1902) + 158.2$ in intercept form. $y = 13.6x - 25{,}709$

b. Write $y = -5.2x + 15$ in point-slope form using $x = 10$ as the first coordinate of the point. $y = -37 - 5.2(x - 10)$

5. Consider the point-slope equation $y = -3.5 + 2(x + 4.5)$.

a. Name the point used to write this equation. $(-4.5, -3.5)$

b. Write an equivalent equation in intercept form. $y = 2x + 5.5$

c. Factor your answer to 5b and name the x-intercept. $y = 2(x + 2.75)$; the x-intercept is -2.75.

d. A point on the line has a y-coordinate of 16.5. Find the x-coordinate of this point and use this point to write an equivalent equation in point-slope form.

e. Explain how you can verify that all four equations are equivalent. Answers will vary. Possible methods are graphing, using a calculator table, and putting all equations in intercept form.

Helping with the Exercises

1. $x_2 = 4$

3. Line a has slope -1, y-intercept 1, and equation $y = 1 - x$. Line b has slope 2, y-intercept -2, and equation $y = -2 + 2x$.

Exercise 5 Some students may have missed the idea of factoring from Lesson 4.4, Exercise 7. Realizing that the x-intercept is the number subtracted from x might also be a challenge. **[Ask]** "What value of x makes y equal zero?"

5d. The x-coordinate is 5.5; $y = 16.5 + 2(x - 5.5)$.

6. Show all steps for a symbolic solution to each problem.

a. $4 + 2.8x = 51$ **b.** $38 - 0.35x = 27$

c. $11 + 3(x - 8) = 41$ **d.** $220 - 12.5(x - 6) = 470$

7. **APPLICATION** Suppose Karl bought a used car for $12,600. Each year its value is expected to decrease by $1,350.

a. Write an equation modeling the value of the car over time. Let x represent the number of years Karl owns the car, and let y represent the value of the car in dollars. $y = 12{,}600 - 1{,}350x$

b. What is the slope, and what does it mean in the context of the problem?

c. What is the y-intercept, and what does it mean in the context of the problem?

d. What is the x-intercept, and what does it mean in the context of the problem?

8. Recall the data about heating a pot of water from the investigation in Lesson 4.3. A possible linear model relating the time in seconds, x, to the temperature in °C, y, is $y = 30 + 0.375(x - 36)$.

a. What equation could you solve to find how long it would take before the pot of water reaches 43°C? $43 = 30 + 0.375(x - 36)$

b. Find the approximate time indicated in 8a using a table or graph. $x \approx 71$ s

c. Show a symbolic solution for your equation in 8a. $x = \dfrac{43 - 30}{0.375} + 36 = 70.\overline{6}$

9. **APPLICATION** The table gives the winning heights for the Olympic women's high jump.

a. Find the five-number summaries for the year and height data.

Women's High Jump

Year	Champion	Height (m)
1956	Mildred McDaniel, United States	1.76
1960	Iolanda Balas, Romania	1.85
1964	Iolanda Balas, Romania	1.90
1968	Miloslava Rezková, Czechoslovakia	1.82
1972	Ulrike Meyfarth, West Germany	1.92
1976	Rosemarie Ackerman, East Germany	1.93
1980	Sara Simeoni, Italy	1.97
1984	Ulrike Meyfarth, West Germany	2.02
1988	Louise Ritter, United States	2.03
1992	Heike Henkel, Germany	2.02
1996	Stefka Kostadinova, Bulgaria	2.05
2000	Yelena Yelesina, Russia	2.01
2004	Yelena Yelesina, Russia	2.06

(International Olympic Committee, in *The World Almanac and Book of Facts 2004,* p. 869) [Data sets: **JMPYR, JMPHT**]

Yelena Yelesina was the first Russian woman to win the Olympic high jump title.

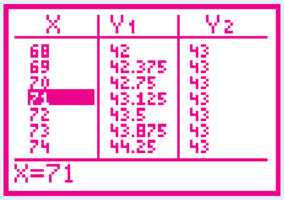
Exercise 6 You might ask students to give the reason for each step as a review of the algebraic properties.

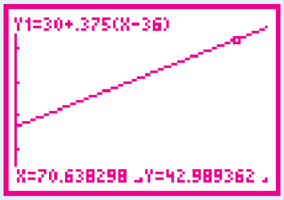
7b. $-1{,}350$; the car's value decreases by $1,350 each year.

7c. 12,600; Karl paid $12,600 for the car.

7d. $9\frac{1}{3}$; in $9\frac{1}{3}$ years the car will have no monetary value.

8b.





[0, 80, 10, 0, 50, 10]

Exercise 9 Rounding the numerical value of the slope will affect the answer to 9e.

9a. 1956, 1966, 1980, 1994, 2004; 1.76, 1.875, 1.97, 2.025, 2.06

6a. $4 + 2.8 = 51$

$2.8x = 51 - 4 = 47$

$x = \dfrac{47}{2.8} \approx 16.8$

6b. $38 - 0.35x = 27$

$-0.35x = 27 - 38 = -11$

$x = \dfrac{-11}{-0.35} \approx 31.4$

6c. $11 + 3(x - 8) = 41$

$3(x - 8) = 41 - 11 = 30$

$x - 8 = \dfrac{30}{3} = 10$

$x = 10 + 8 = 18$

6d. $220 - 12.5(x - 6) = 470$

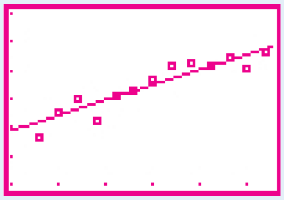
$-12.5(x - 6) = 470 - 220 = 250$

$x - 6 = \dfrac{250}{-12.5} = -20$

$x = -20 + 6 = -14$

9b. The Q-points are (1966, 1.875) and (1994, 2.025).

9d. Answers will vary. There are more points above the line than below the line.



[1950, 2005, 10, 1.6, 2.2, 0.1]

Exercise 10 **[Link]** You may want to have students do research on minimum wage. Some questions to research are these: Why was a minimum-wage law enacted? Why doesn't the minimum wage change every year? Does the minimum-wage law apply to all professions?

10a. $y = 2.25 + 0.13(x - 1976.5)$ or $y = 4.025 + 0.13(x - 1990.5)$

10b. The slope means the minimum hourly wage increased approximately \$0.13 per year.

10c. Using the equation $y = 2.25 + 0.13(x - 1976.5)$, the prediction is \$6.61; if the other equation is used, the prediction is \$6.56.

10d. Using either equation from 10a, the prediction is 1967.

11. Answers will vary. Possible answers:

11a. In an equation written as $y = a + bx$, b is the slope and a is the y-intercept.

11b. If the points are (x_1, y_1) and (x_2, y_2), then the slope of the line is given by the equation $\frac{y_2 - y_1}{x_2 - x_1} = b$. The equation of the line is $y = y_1 + b(x - x_1)$.

b. Name the Q-points for this data set.

c. Write an equation for the line through the Q-points. $y = 1.875 + 0.00536(x - 1966)$ or $y = 2.025 + 0.00536(x - 1994)$

d. Graph the line and the data, and explain whether or not you think this line is a good model for the data pattern.

e. Predict the winning height for the year 2012. Using $y = 1.875 + 0.00536(x - 1966)$, the prediction is 2.12 m.

10. **APPLICATION** This table shows the federal minimum hourly wage for 1974–1997.

a. Find the line of fit based on Q-points.

b. Give the real-world meaning of the slope.

c. Use your model to predict the minimum hourly wage for 2010.

d. Estimate when the minimum hourly wage was \$1.00.

United States Minimum Wage

Year x	Hourly minimum y	Year x	Hourly minimum y
1974	\$1.90	1980	\$3.10
1975	\$2.00	1981	\$3.35
1976	\$2.20	1990	\$3.80
1977	\$2.30	1991	\$4.25
1978	\$2.65	1996	\$4.75
1979	\$2.90	1997	\$5.15

(Bureau of Labor Statistics, *www.bls.gov*)

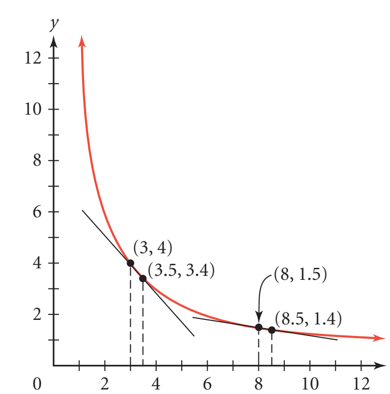
11. Explain how to find the equation of a line when you know

a. The slope and the y-intercept.

b. Two points on that line.

TAKE ANOTHER LOOK

Is rate of change the same as slope? For linear equations, you've seen that it is. But what about curves? You've studied inverse variations, whose equations have the form $y = \frac{k}{x}$. Let's look at the equation $y = \frac{12}{x}$ and its graph.



(3, 4) is a point on the curve. Let's choose another nearby point. Substituting 3.5 for x in the equation, you get $y \approx 3.4$. Using the points (3, 4) and (3.5, 3.4) in the formula for slope, you get

$$b = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3.4 - 4}{3.5 - 3} = \frac{-0.6}{0.5} = -1.2$$

We can say that the *average* rate of change for $y = \frac{12}{x}$ on the interval $x = 3$ to $x = 3.5$ is -1.2. But -1.2 is not the "slope" of $y = \frac{12}{x}$. Instead, it is the slope of the *straight* line through the two points (3, 4) and (3.5, 3.4). Is the average rate of change on the x-interval from 3 to 3.25 the same as from 3.25 to 3.5?

Try points on the "wings" of the curve. For instance, (8, 1.5) is on the curve and so is (8.5, 1.4). Again, the y-coordinate is approximate. What is the average rate of change between these points? The x-interval is the same as for the points (3, 4) and (3.5, 3.4), but is the rate of change the same? What does this tell you? What *straight* line through (8, 1.5) has slope equal to the average rate of change on the interval $x = 8$ to $x = 8.5$?

Take Another Look

The rate of change of a curve (other than a straight line) is not constant. In general, you can't find the slope of a curve at a point by finding the slope of a line between two points on the curve, no matter how close together those points are. The average rate of change over the x-interval from 3 to 3.25 will not be the same as over the interval from 3.25 to 3.5.

The rate of change between the points (3, 4) and (3.5, 3.4) is -1.2 and between (8, 1.5) and (8.5, 1.4) is -0.2. This tells us that the rate of change of the y-values is slower on the wings than at the portion of the graph nearest the origin. The equation of the line through the points (8, 1.5) and (8.5, 1.4) is $y = 1.5 - 0.2(x - 8)$.

Assessing What You've Learned

PERFORMANCE ASSESSMENT

This chapter has been about writing equations for lines, recognizing equivalent equations written in different forms, and fitting lines to data. So, assessing what you've learned really means checking to see if you can write the equation for a given line in one or more forms, if you can find an equivalent equation for the one you've already written, and if you can write an equation for a line that looks like a good fit for a given set of data. Can you do one of the investigations in this chapter on your own? Can you verify whether two equations are equivalent? Showing that you can do tasks like these is sometimes called "performance assessment."

Review the Investigations Equivalent Equations in Lesson 4.4 and Life Expectancy in Lesson 4.5. Identify the equivalent equations in Step 6 of Investigation 4.4, and reconstruct your work in Steps 1–4 of Investigation 4.5. See if the skills you learned in these investigations have become easier for you. Get help with any part of the processes you're not sure of.

As a classmate, parent, or your teacher watches, convert an equation in point-slope form to intercept form. Explain each step, and show how you might verify that the two equations are equivalent using a graph or table. Then, find an equation that is a good fit for a set of data, using any method you like. Show that your equation is a good fit, and use your equation to make a prediction.

UPDATE YOUR PORTFOLIO Choose a piece of work from this chapter to add to your portfolio. Describe the work in a cover sheet, giving the objective, the result, and what you might have done differently.

WRITE IN YOUR JOURNAL What have you enjoyed more in studying algebra—the numbers, symbols, graphs, and other abstract ways of describing relationships, or the concrete applications and examples that show how people use these ideas in the real world?

Do you find it interesting that a single linear relationship can be described in so many ways, or does that add confusion for you?

GIVE A PRESENTATION Research a topic of interest to you that involves two kinds of numerical data. Present the data in a table, make a scatter plot, and describe the pattern of the points. If the data points show a linear pattern, tell how to find a line of fit for the data set and why that line is useful.

ASSESSING

Choose three or four constructive assessment items from the Assessment Resources. Use one of the chapter tests, or create your own test.

FACILITATING SELF-ASSESSMENT

To help students complete the portfolio described in Assessing What You've Learned, suggest that they consider for evaluation their work on Lesson 4.1, Exercise 7; Lesson 4.2, Exercises 5 and 7; Lesson 4.3, Exercise 10; Lesson 4.4, Exercises 12 and 13; Lesson 4.5, Exercise 8; Lesson 4.6, Exercise 4; and Lesson 4.7, Exercise 4.